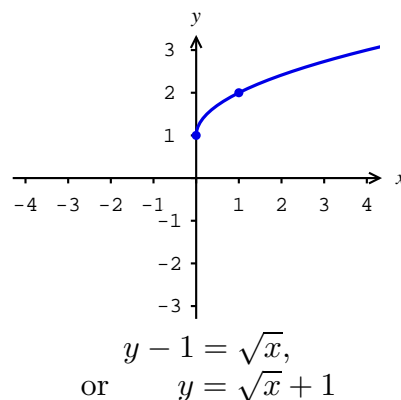
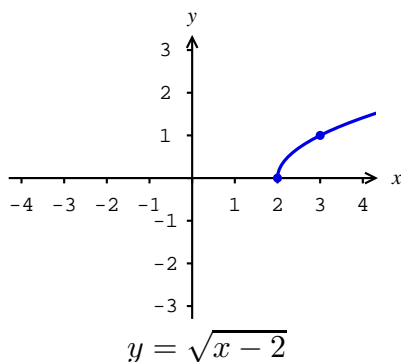
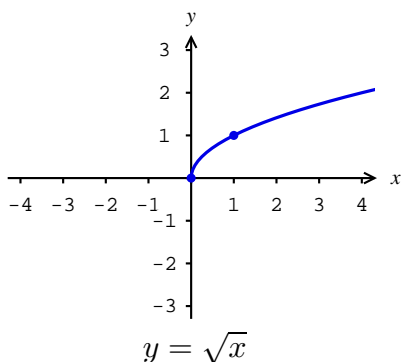
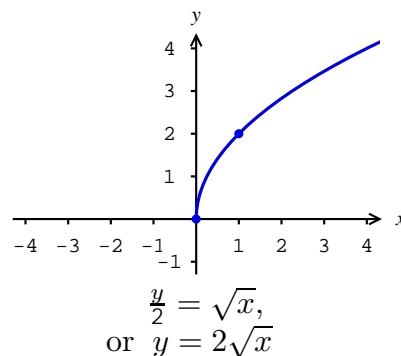
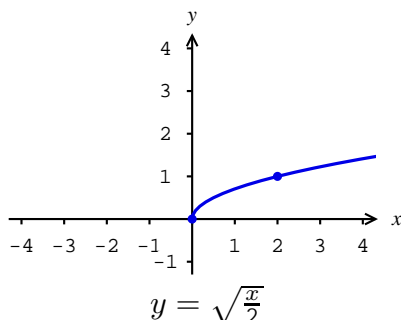
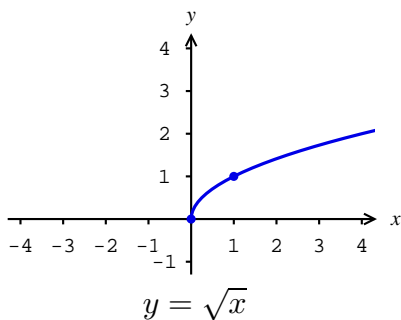
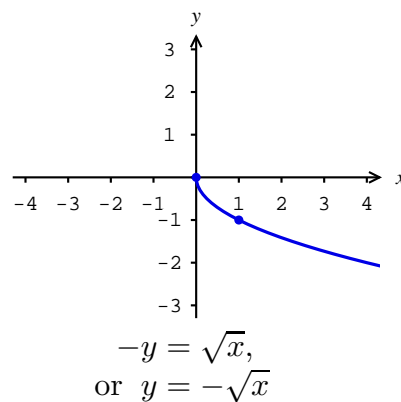
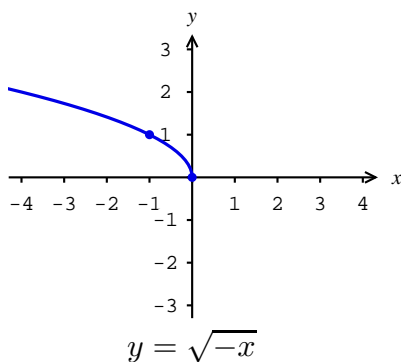
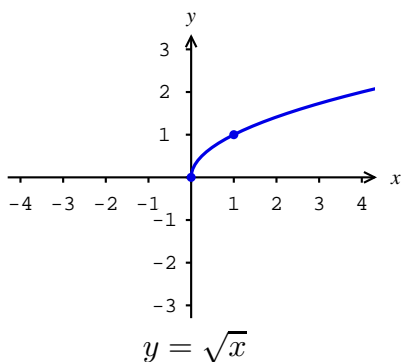


Transformations on the graphs of equations

To understand how changes to the equation change that equation's graph, let's look at some examples using $y = \sqrt{x}$. Check for yourself that the coordinates of the points marked with the dot (\bullet) on each graph satisfy the given equation.



Since the equation $y = \sqrt{x}$ has the form $y = f(x)$, the equations in the third column that result from replacing y by some other expression could also have been obtained instead by replacing the function $f(x)$ by some other function.

These figures illustrate the following rules. Suppose $E(x, y)$ is an equation in x and y .

Reflections:

1. The graph of $E(-x, y)$ is obtained by reflecting the graph of $E(x, y)$ across $x = 0$.
2. The graph of $E(x, -y)$ is obtained by reflecting the graph of $E(x, y)$ across $y = 0$.

Scaling:

3. The graph of $E(\frac{x}{a}, y)$ is obtained by stretching the graph of $E(x, y)$ horizontally by a factor of a .
4. The graph of $E(x, \frac{y}{b})$ is obtained by stretching the graph of $E(x, y)$ vertically by a factor of b .

Translations:

5. The graph of $E(x - h, y)$ is obtained by shifting the graph of $E(x, y)$ h units right.
6. The graph of $E(x, y - k)$ is obtained by shifting the graph of $E(x, y)$ k units up.

In 5. and 6., “right” and “up” are interpreted according the signs of h and k . For instance, -6 units to the right means 6 units left.

In the special case that the $E(x, y)$ has the form $y = f(x)$, then these rules tell us how changes to the function $f(x)$ affect its graph.

Reflections:

- 1'. The graph of $f(-x)$ is obtained by reflecting the graph of $f(x)$ across $x = 0$.
- 2'. The graph of $-f(x)$ is obtained by reflecting the graph of $f(x)$ across $y = 0$.

Scaling:

- 3'. The graph of $f(\frac{x}{a})$ is obtained by stretching the graph of $f(x)$ horizontally by a factor of a .
- 4'. The graph of $bf(x)$ is obtained by stretching the graph of $f(x)$ vertically by a factor of b .

Translations:

- 5'. The graph of $f(x - h)$ is obtained by shifting the graph of $f(x)$ h units right.
- 6'. The graph of $f(x) + k$ is obtained by shifting the graph of $f(x)$ k units up.