No notes, books, electronic devices, or outside materials of any kind.

Read each problem carefully and simplify your answers.

Unless otherwise indicated, supporting work will be required on every problem worth more than 3 points.

You are expected to know the values of all trig functions at multiples of $\pi/4$ and of $\pi/6$.

1a(10 pts). State the precise, ε-δ definition of what it means for $\lim_{x \to c} g(x) = K$.

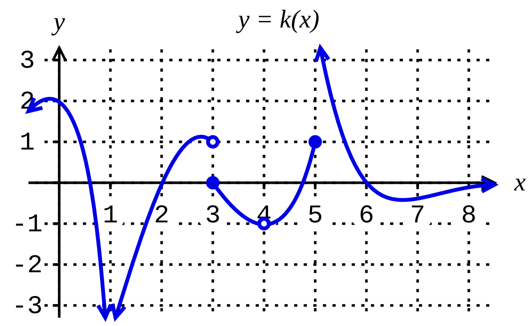
1b(9 pts). Write an ε-δ proof of the fact that $\lim_{x \to -2}(4 - 3x) = 10$.

2(32 pts). Evaluate the limit or briefly explain why it does not exist.

a. $\displaystyle\lim_{x \to -2} \frac{x^2 - 4}{2x^2 + 3x - 2}$ b. $\displaystyle\lim_{x \to 5} \frac{\sqrt{3x+1} - 4}{x - 5}$ c. $\displaystyle\lim_{x \to 3^+} \frac{2 - x}{x - 3}$

d. $\displaystyle\lim_{x \to -\infty} \frac{3x^2 + 2}{x^3 + 2x}$ e. $\displaystyle\lim_{x \to \infty} \frac{e^{2x} - 3e^{-x}}{3e^{2x} + e^{-x}}$ f. $\displaystyle\lim_{x \to 1^+} \ln(x - 1)$

3. (Supporting work is not required on this problem.) See the graph of $k(x)$.



a(4 pts). At which x-values, if any, is $k(x)$ discontinuous?

b(2 pts). At which x-values, if any, is $k(x)$ discontinuous but continuous from the left?

3, continued(11 pts). Find the following. Your answer to each should be ∞, $-\infty$, a number, or "DNE" (for "does not exist").

c. $\displaystyle\lim_{x \to 1} k(x)$ d. $\displaystyle\lim_{x \to 5} k(x)$ e. $k(1)$ f. $k(4)$

g. $\displaystyle\lim_{x \to 3^-} k(x)$ h. $\displaystyle\lim_{x \to 6} k(x)$ i. $k(2)$ j. $k(6)$

k. $\displaystyle\lim_{x \to 4} k(x)$ l. $\displaystyle\lim_{x \to \infty} k(x)$ m. $k(3)$

4(7 pts). On the axes below, sketch the graph of a continuous function $v(x)$ on the the interval $[0, 4]$ for which $v(0) = v(2) = v(4) = 0$, $v'(2) = -1$, and $v'(1) = v'(3) = 0$.

5. Let $g(x) = \frac{1}{3x+2}$. The point $(0, \frac{1}{2})$ is on the graph of $g(x)$.

5a(5 pts). Find the slope of the secant line joining the point $(0, \frac{1}{2})$ and the point on the graph of $g(x)$ corresponding to $x = 1$.

5b(15 pts). Use the definition of the derivative to find $g'(a)$.
You can use shortcut methods from Chapter 3 to check your work, but to receive credit on this problem, you must calculate the derivative from its definition as a limit.

5c(5 pts). Find an equation of the line tangent to the graph of $g(x)$ at the point $(0, \frac{1}{2})$.

1a(10 pts).(Source: Students were told in class to be prepared to state this definition from section 2.4.) $\lim_{x\to c} g(x) = K$ means that, for any positive number ε, there's a corresponding positive number δ with the property that $|g(x) - K| < \varepsilon$ whenever $0 < |x - c| < \delta$.
In this definition and the proof below, **wording matters**. Small changes in wording can significantly change the meaning of what you've written.
1b(9 pts).(Source: 2.4.20, 2.4.re7) Here's the thinking I did before writing my proof:

$$|4 - 3x - 10| = | - 3x - 6| = | - 3||x + 2| = 3|x + 2|.$$

To acheive $3|x + 2| < \varepsilon$, just make sure that $|x + 2| < \frac{1}{3}\varepsilon = \delta$.
Proof: Suppose that $\varepsilon > 0$, and choose $\delta = \frac{1}{3}\varepsilon$. Then

$$|4 - 3x - 10| = | - 3x - 6| = | - 3||x + 2| = 3|x + 2| < 3\delta = \varepsilon$$

whenever $0 < |x + 2| < \delta$, as desired.
2a(6 pts).(Source: 2.3.15) Factor and cancel: $\lim_{x\to -2} \frac{(x-2)(x+2)}{(2x-1)(x+2)} = \lim_{x\to -2} \frac{x-2}{2x-1} = \frac{-4}{-5} = \frac{4}{5}$.
2b(8 pts).(Source: 2.3.21,22) Rationalize the numerator:

$$\begin{aligned}\frac{\sqrt{3x+1} - 4}{x - 5} \cdot \frac{\sqrt{3x+1} + 4}{\sqrt{3x+1} + 4} &= \frac{\sqrt{3x+1}^2 - 4^2}{(x-5)(\sqrt{3x+1} + 4)} = \frac{(3x+1) - 16}{(x-5)(\sqrt{3x+1} + 4)} \\ &= \frac{3x + 1 - 15}{(x-5)(\sqrt{3x+1} + 4)} = \frac{3(x-5)}{(x-5)(\sqrt{3x+1} + 4)}\end{aligned}$$

which, as long as $x \neq 5$, equals

$$\frac{3}{\sqrt{3x+1} + 4}.$$

Therefore the limit in 2b is the same as

$$\lim_{x\to 5} \frac{3}{\sqrt{3x+1} + 4} = \frac{3}{\sqrt{16} + 4} = \frac{3}{8}.$$

2c(4 pts).(Source: 2.2.31,32) As $x \to 3^+$, $\frac{2-x}{x-3}$ looks like "$\frac{-1}{0}$," indicating blow-up. Examine the signs. The numerator is near -1 and therefore must be negative. When $x > 3$, $x - 3 > 0$, so the fraction is $\frac{-}{+} = -$ and so $\lim_{x\to 3^+} \frac{2-x}{x-3} = -\infty$.
2d(4 pts).(Source: 2.6.18,31) The limit as $x \to \pm\infty$ of a rational function is the same as the limit of the ratio of its lead terms:

$$\lim_{x\to -\infty} \frac{3x^2 + 2}{x^3 + 2x} = \lim_{x\to -\infty} \frac{3x^2}{x^3} = \lim_{x\to \infty} \frac{3}{x} = 0.$$

2e(8 pts).(Source: 2.6.36) The limit looks like "$\frac{\infty}{\infty}$". Factor out the dominant terms of the numerator and denominator:

$$\frac{e^{2x} - 3e^{-x}}{3e^{2x} + e^{-x}} = \frac{e^{2x}(1 - 3e^{-3x})}{e^{2x}(3 + e^{-3x})} = \frac{1 - 3e^{-3x}}{3 + e^{-3x}}$$

$\lim_{x\to\infty} e^{-3x} = 0$, so

$$\lim_{x\to\infty} \frac{1-3e^{-3x}}{3+e^{-3x}} = \frac{1}{3}.$$

2f(2 pts).(Source: 2.2.35) Recall from the graph of $\ln x$ that $\lim_{x\to 0^+} \ln x = -\infty$. As $x \to 1^+$ in 2f, $(x-1) \to 0^+$, and so $\lim_{x\to 1^+} \ln(x-1) = -\infty$. (See also Example 11, p. 62 of the text.)

3.(Source: 2.2.4-9, 2.5.3, 2.6.3,4)
a(4 pts). The x-values where $k(a)$ fails to equal $\lim_{x\to a} k(x)$ are 1, 3, 4, and 5.
b(2 pts). At 1, 3, 4, and 5, k is continuous from the left only at 5. There, $k(5) = 1 = \lim_{x\to 5^-} k(x)$.
Remaining parts are worth 1 point each.

c. $-\infty$	d. DNE	e. DNE	f. DNE
g. 1	h. 0	i. 0	j. 0
k. -1	l. 0	m. 0	

4(7 pts).(Source: 2.7.24) You can see a correct graph of $v(x)$ at
https://www.desmos.com/calculator/mhkrll1b6h

5a(5 pts).(Source: 2.1.3) $g(1) = \frac{1}{5}$, and the slope of the line joining the points $(0, \frac{1}{2})$ and $(1, \frac{1}{5})$ is $\dfrac{\frac{1}{5}-\frac{1}{2}}{1-0}$, or $-\frac{3}{10}$.

5b(15 pts).(Source: 2.7.8,33, 2.7.more.1k)

$$\begin{aligned} g'(a) &= \lim_{h\to 0} \frac{\frac{1}{3(a+h)+2} - \frac{1}{3a+2}}{h} = \lim_{h\to 0} \frac{\frac{1}{3(a+h)+2} - \frac{1}{3a+2}}{h} \cdot \frac{(3(a+h)+2)(3a+2)}{(3(a+h)+2)(3a+2)} \\ &= \lim_{h\to 0} \frac{(3a+2)-(3(a+h)+2)}{h(3(a+h)+2)(3a+2)} \\ &= \lim_{h\to 0} \frac{3a+2-3a-3h-2}{h(3(a+h)+2)(3a+2)} \\ &= \lim_{h\to 0} \frac{-3h}{h(3(a+h)+2)(3a+2)} = \lim_{h\to 0} \frac{-3}{(3(a+h)+2)(3a+2)} = \frac{-3}{(3a+2)^2} \end{aligned}$$

5c(5 pts).(Source: 2.7.8) The line with slope $g'(0) = -\frac{3}{4}$ passing through the point $(0, \frac{1}{2})$ is $y - \frac{1}{2} = -\frac{3}{4}x$.