MATH 120–04,11 (Kunkle), Exam 4 Name: ____________________

100 pts, 75 minutes Apr 11, 2024

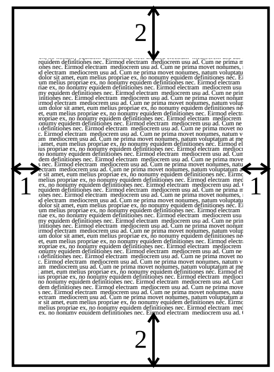
No notes, books, electronic devices, or outside materials of any kind.

Read each problem carefully and simplify your answers.

Unless otherwise indicated, supporting work will be required on every problem worth more than 2 points.

You are expected to know the values of all trig functions at multiples of $\pi/4$ and of $\pi/6$.

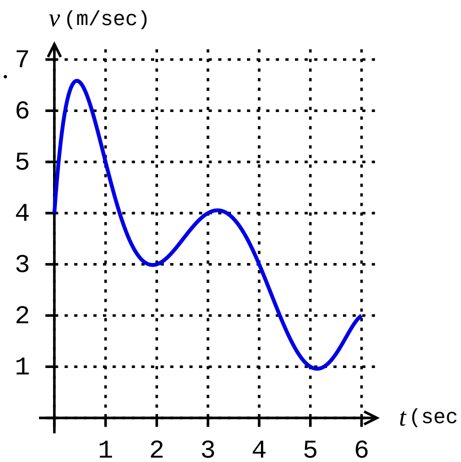
1(18 pts). The top and bottom margins of a poster are each 2 inches, and the side margins are each 1 inch. If the area of printed material on the poster must be 8 square inches, what is the smallest possible total surface area of the poster?



2(28 pts). If $g(x) = \frac{x^2}{x^2-4}$, then $g'(x) = \frac{-8x}{(x^2-4)^2}$ and $g''(x) = \frac{24x^2+32}{(x^2-4)^3}$.

a. Find the domain of $g(x)$.
b. Find the x- and y-intercepts of the graph of $g(x)$.
c. Find equations of all asymptotes of the graph of $g(x)$.
d. Is $g(x)$ even, odd, or neither?
e. Find the interval(s) on which $g(x)$ is increasing.
f. Find the interval(s) on which the graph of $g(x)$ is concave up.
g. Illustrate properties discovered in a-f with a graph of $g(x)$.

3(6 pts). The graph of the velocity a moving object is shown at right. Estimate the distance traveled by the object between times $t = 0$ and $t = 6$ with a Riemann sum using $n = 3$ subintervals and their midpoints.



4(15 pts). Find the general antiderivative of the given function.

a. $2\sin x + \sec x\,(\sec x - \tan x)$ b. $\dfrac{3+x^2}{\sqrt{x}} - \dfrac{3}{x} + \dfrac{1}{3}$

5(7 pts). Find the indicated derivative.

a. $\dfrac{d}{dx}\displaystyle\int_2^x \frac{\sin t}{t^2+1}\,dt$ b. $\dfrac{d}{dx}\displaystyle\int_1^{3x} \frac{\sin t}{t^2+1}\,dt$

6(10 pts). Suppose $\int_1^2 r(x)\,dx = -2$ and $\int_1^5 r(x)\,dx = 8$. Find the following.

a. $\displaystyle\int_5^1 r(x)\,dx$ b. $\displaystyle\int_4^4 r(x)\,dx$ c. $\displaystyle\int_2^5 2r(x)\,dx$

7(16 pts). Evaluate the definite integral.

a. $\displaystyle\int_1^3 (x-1)(x+3)\,dx$ b. $\displaystyle\int_0^{\pi/3} (e^x + \cos\theta)\,d\theta$

1(18 pts).(Source: 4.7.35) Let x and y stand for the base and height respectively of the printed part of the poster (that is, the smaller rectangle in the picture). Then the entire poster has dimensions $x+2$ by $y+4$. The total area of the poster is $f=(x+2)(y+4)$ and the printed area is $xy=8$. Solve to find $y=8x^{-1}$ and substitute this into f:

$$f = xy + 2y + 4x + 8 = 16 + 16x^{-1} + 4x$$

Note that for any positive x-value there's a corresponding value of y for which $xy=8$. Therefore, we wish to minimize $f(x)$ on the interval $0 < x < \infty$.
Find critical points by setting $f'(x)=0$:

$$-16x^{-2} + 4 = 0 \quad \Longrightarrow \quad x^2 = 4 \quad \Longrightarrow \quad x = 2$$

(since x can't be negative). Factor $f'(x) = 4x^{-2}(-4+x^2)$ and make its sign chart on $(0,\infty)$:

$f'(x) = \underbrace{4x^{-2}}_{+}(-4+x^2)$:	$- - - - - 0 + + + + +$
x :	0 2

Since f decreases on $(0,2)$ and increases on $(2,\infty)$, its absolute minimum on $(0,\infty)$ occurs at $x=2$. The minimum area of the poster is $f(2) = 16 + \frac{16}{2} + 4 \cdot 2 = 32$.

2.(Source: 4.5.13, 15)

a(2 pts). g is defined for all x except where $x^2-4=0$ at $x=\pm 2$. In interval notation, the domain is $(-\infty,-2) \cup (-2,2) \cup (2\infty)$.

b(2 pts). Only x- or y-intercept is the point $(0,0)$.

c(3 pts). $|y| \to \infty$ as $x \to \pm 2$, so $x=2$ and $x=-2$ are V.A.. $y \to 1$ as $x \to \pm\infty$, so H.A. is $y=1$.

d(3 pts). Because $(-x)^2 = x^2$, $g(-x) = g(x)$. That is, g is even. (Consequently, its graph is symmetric across the y-axis.)

e(4 pts). Sign chart for $g'(x)$:

$-8x$:	$+++++++++\,0---------$
$(x^2-4)^2$:	$++++\,0+++++++++\,0++++$
$\frac{-8x}{(x^2-4)^2}$:	$+++$DNE$+++\,0\,---$DNE$---$
x :	-2 0 2

Conclusion: g increases on $(-\infty,-2)$ and on $(-2,0)$.

f(5 pts). Sign chart for $g''(x)$:

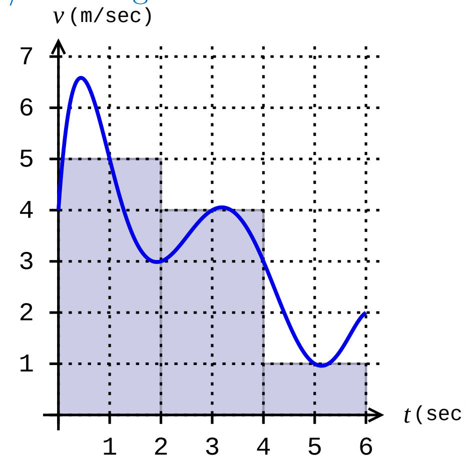
$24x^2+32$:	$+++++++++++++++++++$
$(x^2-4)^3$:	$++++\,0---------\,0++++$
$\frac{-8x}{(x^2-4)^2}$:	$+++$DNE$-------$DNE$+++$
x :	-2 2

Conclusion: graph of g is concave up on $(-\infty, -2)$ and on $(2, \infty)$.

g(9 pts). See graph at https://www.desmos.com/calculator/ebk3ifstgm

3(6 pts).(Source: 5.1.1,2,17,18) Each subinterval has length $\Delta t = 2$, their midpoints are $t = 1$, 3, and 5, and the Riemann sum is

$$5 \cdot 2 + 4 \cdot 2 + 1 \cdot 2 = 20\text{m}.$$



See summary of antidifferentiation rules in section 4.9 of the review notes found at http://kunklet.people.cofc.edu/MATH120/120review.pdf

4a(7 pts).(Source: 4.9.17,37) Distribute $\sec x$ across the difference $(\sec x - \tan x)$. General antiderivative of $2\sin x + \sec^2 x - \sec x \tan x$ is $-2\cos x + \tan x - \sec x + C$.

4b(8 pts).(Source: 4.9.13, 15) Rewrite $\frac{3+x^2}{\sqrt{x}}$ as $3x^{-1/2} + x^{3/2}$. General antiderivative is $3 \cdot 2x^{1/2} + \frac{2}{5}x^{5/2} - 3\ln|x| + \frac{1}{3}x + C$.

5.(Source: 5.3.9,17) FTC part 1, p. 394, tells us that $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$ wherever f is continuous.

a(2 pts). $\dfrac{d}{dx}\displaystyle\int_2^x \frac{\sin t}{t^2+1}\,dt = \frac{\sin x}{x^2+1}$, since this is continuous everywhere.

b(5 pts). Let's call the integral I and let $u = 3x$. Then

$$\frac{dI}{dx} = \frac{dI}{du}\frac{du}{dx} = \frac{\sin u}{u^2+1}(3x)' = \frac{\sin(3x)}{(3x)^2+1} \cdot 3 = \frac{3\sin(3x)}{9x^2+1}$$

6a(2 pts).(Source: 5.2.42) $\int_5^1 r(x)\,dx = -\int_1^5 r(x)\,dx = -8$.

6b(2 pts).(Source: 5.2.41) $\int_4^4 r(x)\,dx = 0$.

6c(6 pts).(Source: 5.2.48-49)

$$\int_2^5 2r(x)\,dx = 2\left(\int_2^5 r(x)\,dx\right) = 2\left(\int_1^5 r(x)\,dx - \int_1^2 r(x)\,dx\right) = 2\,(8 - (-2)) = 20.$$

7. FTC part 2, p. 396, tells us that if $F'(x) = f(x)$ for all x in $[a, b]$ and if $f(x)$ is continuous on $[a, b]$, then $\int_a^b f(x)\,dx = F(b) - F(a)$.

7a(8 pts).(Source: 5.3.27) $\int_1^3 (x-1)(x+3)\,dx = \int_1^3 (x^2+2x-3)\,dx = (\frac{1}{3}x^3 + x^2 - 3x)\Big|_1^3$
$= (\frac{1}{3}3^3 + 3^2 - 3 \cdot 3) - (\frac{1}{3} + 1 - 3) = 10 + \frac{2}{3}$, or $\frac{32}{3}$.

7b(8 pts).(Source: 5.3.25)

$$\int_0^{\pi/3} (e^x + \cos\theta)\,d\theta = (e^\theta + \sin\theta)\Big|_0^{\pi/3} = (e^{\pi/3} + \sin\tfrac{\pi}{3}) - (e^0 + \sin 0) = e^{\pi/3} + \tfrac{\sqrt{3}}{2} - 1.$$

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