MATH 120–04 (Kunkle), Quiz 2   Name: ______________________
10 pts, 10 minutes   Jan 25, 2024

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1 (10 pts). Evaluate the limit, if it exists. Supporting work is required; one-word answers are not acceptable for full credit.

a. $\displaystyle\lim_{x\to 4} \frac{x^2-2x-8}{x^2-9x+20}$ b. $\displaystyle\lim_{x\to 5^-} \frac{x^2-2x-8}{x^2-9x+20}$ c. $\displaystyle\lim_{x\to 5^+} \frac{x^2-2x-8}{x^2-9x+20}$

Solution:

1a.(Source: 2.3.15,16) Factor and cancel:

$$\lim_{x\to 4} \frac{(x-4)(x+2)}{(x-4)(x-5)} = \lim_{x\to 4} \frac{x+2}{x-5} = \frac{6}{-1} = -6.$$

1b.(Source: 2.3.14) $\displaystyle\lim_{x\to 5^-} \frac{x+2}{x-5}$ looks like "$\frac{7}{0}$," indicating blow-up. Examine the signs. The numerator is near 7 and therefore must be positive. When $x < 5$ as in part b., $x - 5 < 0$, so the fraction is $\frac{+}{-} = -$ and so

$$\lim_{x\to 5^-} \frac{x+2}{x-5} = -\infty.$$

1c.(Source: 2.3.14) Continuing, when $x > 5$, as in part c., $x - 5 > 0$, so the fraction is $\frac{+}{+} = +$ and so

$$\lim_{x\to 5^+} \frac{x+2}{x-5} = \infty.$$

MATH 120–11 (Kunkle), Quiz 2 Name: ______________________________

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1 (10 pts). Find the x-values at which the function

$$f(x) = \begin{cases} x & \text{if } x < -1, \\ 1 - x^2 & \text{if } -1 \le x \le 1, \text{ and} \\ 1/x & \text{if } x > 1. \end{cases}$$

is discontinuous. At which of these numbers is f continuous from the right, from the left, or neither? Supporting work is required, including a sketch the graph of f.

Solution:

1.(Source: 2.5.41-43)

The polynomials x and $1 - x^2$ are continuous everywhere, and the rational function $1/x$ is continuous on $[1, \infty)$ (in fact, it is continuous at all $x \neq 0$), so the only place f could be discontinuous is at the "breakpoints" $x = -1$ and $x = 1$. At these, compare the function value with the one-sided limits:

$$\lim_{x \to -1^-} f(x) = \lim_{x \to -1^-} x = -1 \qquad \lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} 1 - x^2 = 0$$

$$\lim_{x \to -1^+} f(x) = \lim_{x \to -1^+} 1 - x^2 = 0 \qquad \lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} \frac{1}{x} = 1$$

$$f(-1) = 0 \qquad f(1) = 0$$

At both $x = 1$ and $x = -1$, the one-sided limits disagree, so both $\lim_{x \to 1} f(x)$ and $\lim_{x \to -1} f(x)$ fail to exist. We conclude that f is discontinuous at $x = 1$ and at $x = -1$.

f is continuous from the right at $x = -1$, because $\lim_{x \to -1^+} f(x) = 0 = f(-1)$.

f is continuous from the left at $x = 1$, because $\lim_{x \to 1^-} f(x) = 0 = f(1)$.

The graph of f consists of pieces of the line $y = x$, the parabola $y = 1 - x^2$, and the graph of $y = 1/x$ (actually a hyperbola). The graph on the left shows these three functions; the graph of f is on the right.