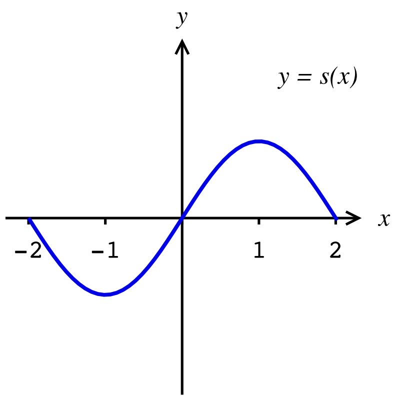
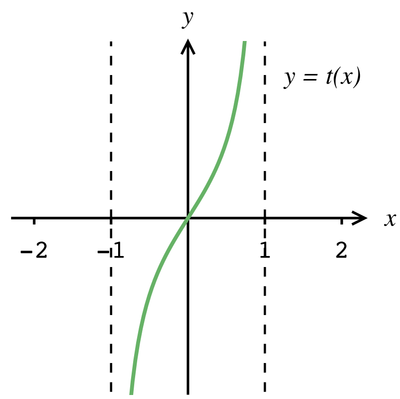
MATH 120–04 (Kunkle), Quiz 3 — Name: ____________________

10 pts, 10 minutes — Feb 8, 2024

---

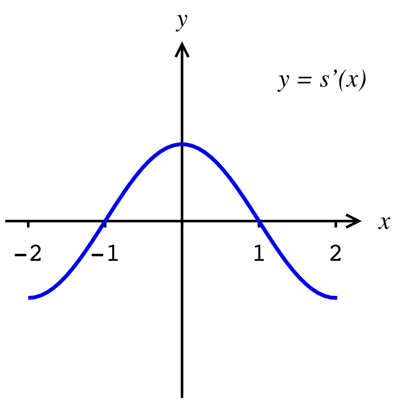
1 (10 pts). The graphs of two functions, $s(x)$ and $t(x)$ appear in the figure below Sketch the graphs of $s'(x)$ and $t'(x)$ on the axes provided.

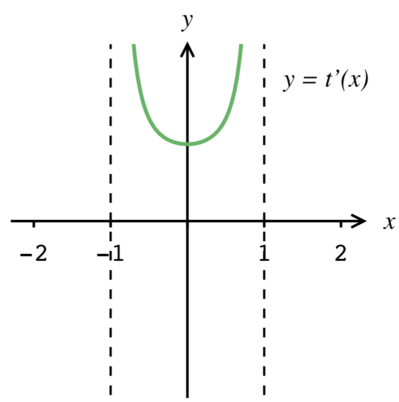




Solution:

1. (Source: 2.8.4-11) Here are the graphs of $s'(x)$ and $t'(x)$.





Necessary components of a correct solution:

At $x =$/On the interval	where the graph of s ...	s' is ...
$x = -1, 1$	is horizontal	is zero.
$(-2,-1)$ and $(1,2)$	has negative slope,	negative.
$(-1,1)$ and $(2,5)$	has positive slope,	positive.

At $x =$/On the interval	where the graph of t ...	t' is ...
$x = -1$ and 1	approaches vertical,	blowing up.
$(-1,1)$	has positive slope,	positive.

In particular, $t'(0)$ is *not* zero.

MATH 120–11 (Kunkle), Quiz 3 Name: ______________________________

10 pts, 10 minutes Feb 8, 2024

---

1 (10 pts). Find the following:

a. $f'(x)$ if $f(x) = \dfrac{2}{\sqrt[3]{x}} + 4^2$

b. $\dfrac{dy}{dx}$ if $y = \dfrac{xe^x}{2x - e^x}$. (You needn't simplify your answer to this.)

c. The equation of the line tangent to the curve $y = 3x^2 - 4x$ at $x = 1$.

Solution:

1a.(Source: 3.1.3,19) Note that $4^2 = 16$ is a constant and that $-\frac{1}{3} - 1 = -\frac{4}{3}$. Since $f(x) = 2x^{-1/3} + 16$, its derivative $f'(x) = -\frac{2}{3}x^{-4/3}$.

1b. (Source: 3.2.23) When more than one rule comes into play, the last operation in the function determines the first rule to use. Ultimately, $y = \frac{xe^x}{2x - e^x}$ is a quotient, so, first of all, the quotient rule tells us

$$\frac{dy}{dx} = \frac{(xe^x)'(2x - e^x) - (2x - e^x)'(xe^x)}{(2x - e^x)^2}.$$

Within the quotient rule, we'll have to use the product rule to differentiant xe^x:

$$\begin{aligned}\frac{dy}{dx} &= \frac{\big(x'e^x + (e^x)'x\big)(2x - e^x) - (2x - e^x)'(xe^x)}{(2x - e^x)^2}\\ &= \frac{(e^x + xe^x)(2x - e^x) - (2 - e^x)(xe^x)}{(2x - e^x)^2}.\end{aligned}$$

Note that $(2x - e^x)$ is **not** a common factor.

You were not required to simplify your answer to part b, but if you did, the result is

$$\frac{dy}{dx} = \frac{e^x(2x^2 - e^x)}{(2x - e^x)^2}$$

1c. (Source: 3.1.33) Calculate $y = 3x^2 - 4x$ and $\frac{dy}{dx} = 3 \cdot 2x - 4 \cdot 1x^0 = 6x - 4$ at $x = 1$ to find that curve passes through the point $(1, -1)$ with slope 2. The tangent line is

$$y + 1 = 2(x - 1).$$

To practice arithmetic with fractions and other basic algebra skills, see

http://kunklet.people.cofc.edu/MATH111/111BootCamp.pdf