MATH 120–11 (Kunkle), Quiz 4   Name: ______________________
10 pts, 10 minutes   Feb 15, 2024

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1 (10 pts). Find the derivative of the given function.

a. $\sqrt{x^2+9e^x}$ b. $e^x \sin x \cos x$ c. $(3x^2-1)^4(x^2+4)^3$ d. $\sec^2(e^{x^3})$

Solution:

1a.(Source: 3.4.6,9) Rewrite the function without the radical and differentiate with the chain rule.

$$\left(\left(x^2+9e^x\right)^{1/2}\right) = \frac{1}{2}\left(x^2+9e^x\right)^{-1/2}\left(x^2+9e^x\right)' = \frac{1}{2}\left(x^2+9e^x\right)^{-1/2}(2x+9e^x).$$

1b.(Source: 3.3.15, 3.3.more.1) The product rule for 3 functions tells us that

$$\begin{aligned}(e^x \sin x \cos x)' &= (e^x)' \sin x \cos x + e^x (\sin x)' \cos x + e^x \sin x (\cos x)' \\ &= e^x \sin x \cos x + e^x \cos x \cos x + e^x \sin x(-\sin x)\end{aligned}$$

or

$$e^x(\sin x \cos x + \cos^2 x - \sin^2 x).$$

1c.(Source: 3.4.17) Use the chain rule within the product rule:

$$\begin{aligned}h'(x) = &\left((3x^2-1)^4\right)'(x^2+4)^3 + (3x^2-1)^4\left((x^2+4)^3\right)' \\ &= 4(3x^2-1)^3(3x^2-1)'(x^2+4)^3 + (3x^2-1)^4 \cdot 3(x^2+4)^2(x^2+4)' \\ &= 4(3x^2-1)^3 \cdot 6x(x^2+4)^3 + (3x^2-1)^4 \cdot 3(x^2+4)^2(2x)\end{aligned}$$

Clean this up a little:

$$= 24x(3x^2-1)^3(x^2+4)^3 + 6x(3x^2-1)^4(x^2+4)^2$$

(done)

In a larger problem, it could be useful to factor in $h'(x)$ in c:

$$\begin{aligned}h'(x) &= 6x(3x^2-1)^3(x^2+4)^2\big(4(x^2+4)+(3x^2-1)\big) \\ &= 6x(3x^2-1)^3(x^2+4)^2(7x^2+15)\end{aligned}$$

1d.(Source: 3.4.41) This part requires multiple applications of the chain rule. From outermost to innermost, the functions in this composition are the squaring function, the secant, the exponential function with base e, and the cubing function:

$$\begin{aligned}\left((\sec(e^{x^3})^2\right)' &= \overbrace{2\sec(e^{x^3})}^{\text{squaring}'}\overbrace{\sec(e^{x^3})\tan(e^{x^3})}^{\text{secant}'}\overbrace{e^{x^3}}^{\text{exp}'}\ \overbrace{3x^2}^{\text{cubing}'} \\ &= 6x^2e^{x^3}\sec^2(e^{x^3})\tan(e^{x^3})\end{aligned}$$