MATH 120–04 (Kunkle), Quiz 6 Name: ______________________________

10 pts, 10 minutes Mar 14, 2024

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1 (10 pts). Find the absolute maximum and minimum values of the function $u(x) = 4x + \frac{1}{x}$ on the interval $[\frac{1}{8}, 1]$.

Solution:

1.(Source: 4.1.53) The critical points of $u(x) = 4x + \frac{1}{x}$ are those x-values where $u(x)$ exists and $u'(x) = 4 - x^{-2}$ is either zero or undefined. Both u and u' are defined at all real numbers except zero, so the only critical points are where $u'(x) = 0$:

$$0 = 4 - x^{-2} \quad \Longrightarrow \quad x^{-2} = 4 \quad \Longrightarrow \quad x^2 = \frac{1}{4}.$$

Now take $\pm$ the square root of both sides:

$$x = \pm\sqrt{\frac{1}{4}} = \pm\frac{1}{2}.$$

The absolute maximum and minimum values of the $u(x)$ on $[\frac{1}{8}, 1]$ can occur only at the endpoints or at critical points in the interval, so we only need to compute and compare the values of $u(x)$ at $\frac{1}{8}$, $\frac{1}{2}$, and 1:

x	$4x + \frac{1}{x}$
$\frac{1}{8}$	$4 \cdot \frac{1}{8} + \frac{1}{1/8} = \frac{1}{2} + 8 = 8.5$
$\frac{1}{2}$	$4 \cdot \frac{1}{2} + \frac{1}{1/2} = 2 + 2 = 4$
1	$4 \cdot 1 + \frac{1}{1} = 5$

Therefore the absolute maximum of u on $[\frac{1}{8}, 1]$ is 8.5 and the absolute minimum is 4.