MATH 120–04,11 (Kunkle), Quiz 7 Name: ____________________
10 pts, 10 minutes Mar 28, 2024

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1. Let $\tau(x) = \dfrac{1}{1+e^x}$. Find the following.

a (1 pts). The domain of τ.
b (2 pts). The horizontal asymptotes of the graph of τ.
c (3 pts). The interval(s) on which τ is decreasing.
d (4 pts). The interval(s) on which the graph of τ is concave down.

Solution:

1a.(Source: 4.5.43) e^x is always positive, so e^x+1 is never zero. The domain of τ is $(-\infty,\infty)$.

1b. $\displaystyle\lim_{x\to-\infty}\frac{1}{1+e^x} = \frac{1}{1+0} = 1$, and $\displaystyle\lim_{x\to\infty}\frac{1}{1+e^x} = \text{“}\frac{1}{\infty}\text{”} = 0$, so $y=1$ and $y=0$ are horizontal asymptotes.

1c. Rewrite $\tau(x)$ as $(1+e^x)^{-1}$. Then by the chain rule, $\tau'(x) = -(1+e^x)^{-2}e^x = \dfrac{-e^x}{(1+e^x)^2}$. e^x is always positive, so $\tau'(x)$ is always negative. Consequently, τ is decreasing on $(-\infty,\infty)$.

1d. Calculate τ'' by the quotient rule and factor:

$$\tau'' = \left(\frac{-e^x}{(1+e^x)^2}\right)' = \frac{-e^x(1+e^x)^2 - (-e^x)2(1+e^x)e^x}{(1+e^x)^4}$$
$$= \frac{e^x(1+e^x)\big((-(1+e^x)+2e^x\big)}{(1+e^x)^4} = \frac{e^x(e^x-1)}{(1+e^x)^3}$$

The factor $\dfrac{e^x}{(1+e^x)^3}$ is always positive, and the factor (e^x-1) is positive if and only if $e^x > 1$ if and only if $x > 0$. Consequently, the graph of τ is concave down on $(-\infty, 0)$.

(done)

The asymptotes, monotonicity, and concavity of $\tau(x)$ show up nicely in this graph:
https://www.desmos.com/calculator/riilq6vfcn