MATH 120–04,11 (Kunkle), Quiz 8 Name: ______________________

10 pts, 10 minutes April 8, 2024

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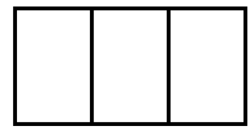
1 (10 pts). A farmer wants to fence in a rectangular pen and divide it in thirds by two additional fences parallel to one of the sides of the rectangle. If she has 64 meters of fence, find the dimensions of the rectangle that will maximize its area (if possible).



Solution:

1.(Source: 4.7.13,18) Let x be the width of the rectangle when drawn as above and y the height. We wish to maximize the area $A = xy$ subject to the constraint that the total length of fence is $2x + 4y = 64$. Rewrite this constraint as $x = 32 - 2y$ and substitute to find that area equals

$$A = (32 - 2y)y = 32y - 2y^2.$$

The smallest allowable value of y is 0. The largest y occurs when $x = 32 - 2y = 0$, so $y = 16$. That is, we wish to maximize the function A on the interval $[0, 16]$.

The absolute maximum of A on $[0, 16]$ must occur at a critical point inside $(0, 16)$ or at an endpoint. To find the critical points of A, on this interval, set $A' = 0$:

$$32 - 4y = 0 \quad \Longrightarrow \quad y = 8.$$

Now compare A at the critical point and the endpoints.

y	$(32 - 2y)y$
0	0
8	128
16	0

The maximum of A occurs when $y = 8$. The dimensions of the rectangle of largest area are $y = 8$ by $x = 32 - 2 \cdot 8 = 16$. **(done)**

You could also determine that A has an absolute max at $y = 8$ by considering the signs of $A' = 4(8 - y)$:

$$\begin{array}{r|l} 4(8-y): & +++++0----- \\ \hline x: & 0 \qquad\qquad 8 \qquad\qquad 16 \end{array}$$

Since A increases for all y in $[0, 8]$ and decreases for all y in $[8, 16]$, it must have an absolute maximum at $y = 8$.