MATH 120–04,11 (Kunkle), Quiz 8
10 pts, 10 minutes

Name: ______________________
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1 (10 pts). Evaluate the integral.

a. $\int \sqrt{r}(r - 1 + \frac{1}{r})\, dr$

b. $\int_0^{1/\sqrt{2}} \frac{1}{\sqrt{1-t^2}}\, dt$

Solution:

1a.(Source: 5.4.10,11) Generally, products are difficult to integrate, so first rewrite the integrand like so:

$$\int r^{1/2}(r - 1 + \frac{1}{r})\, dr = \int (r^{1/2}r^1 - r^{1/2} + \frac{r^{1/2}}{r})\, dr = \int (r^{3/2} - r^{1/2} + r^{-1/2})\, dr$$

By the power rule (Table 1, p. 403), this **indefinite integral** equals

$$= \frac{r^{5/2}}{5/2} - \frac{r^{3/2}}{3/2} + \frac{r^{1/2}}{1/2} + C = \frac{2}{5}r^{5/2} - \frac{2}{3}r^{3/2} + 2r^{1/2} + C.$$

1b.(Source: 5.4.41) Since $\frac{1}{\sqrt{1-t^2}} = \frac{d}{dt}\sin^{-1} t$, the Fundamental Theorem of Calculus, part 2, tells us that this **definite integral** equals

$$\int_0^{1/\sqrt{2}} \frac{1}{\sqrt{1-t^2}}\, dt = \sin^{-1} t\,\Big|_0^{1/\sqrt{2}} = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) - \sin^{-1} 0 = \frac{\pi}{4} - 0 = \frac{\pi}{4}.$$