More problems for section 2.4 of *Calculus, Early Transcendentals* by James Stewart, 8e.

1. Write an ε-δ proof of the following limits.

a. $\lim_{x\to 4} 2x - 3 = 5$

b. $\lim_{x\to -2} 5x + 4 = -6$

c. $\lim_{x\to 1} 6 - 3x = 3$

d. $\lim_{x\to -2} 3 - 4x = 11$

e. $\lim_{x\to 1} \dfrac{(x-1)(3x+5)}{x-1} = 8$

f. $\lim_{x\to 1} \dfrac{(x+2)(2-5x)}{x+2} = 12$

g. $\lim_{x\to -3} \dfrac{10x^2+33x+9}{x+3} = -27$

h. $\lim_{x\to 0} \dfrac{x-6x^2}{x} = 1$

Answers

1a. Suppose $\varepsilon > 0$. Choose $\delta = \frac{\varepsilon}{2}$. Then $|(2x-3)-5| = |2x-8| = 2|x-4| < 2\delta = \varepsilon$ whenever $0 < |x-4| < \delta$, as desired.

1b. Suppose $\varepsilon > 0$. Choose $\delta = \frac{\varepsilon}{5}$. Then $|(5x+4)-(-6)| = |5x+10| = 5|x+2| < 5\delta = \varepsilon$ whenever $0 < |x+2| < \delta$, as desired.

1c. Suppose $\varepsilon > 0$. Choose $\delta = \frac{\varepsilon}{3}$. Then $|(6-3x)-3| = |-3x+3| = 3|x-1| < 3\delta = \varepsilon$ whenever $0 < |x-1| < \delta$, as desired.

1d. Suppose $\varepsilon > 0$. Choose $\delta = \frac{\varepsilon}{4}$. Then $|(3-4x)-11| = |-4x-8| = 4|x+2| < 4\delta = \varepsilon$ whenever $0 < |x+2| < \delta$, as desired.

1e. Suppose $\varepsilon > 0$. Choose $\delta = \frac{\varepsilon}{3}$. Then $|\frac{(x-1)(3x+5)}{x-1} - 8| = |(3x+5)-8| = |3x-3| = 3|x-1| < 3\delta = \varepsilon$ whenever $0 < |x-1| < \delta$, as desired.

1f. Suppose $\varepsilon > 0$. Choose $\delta = \frac{\varepsilon}{5}$. Then $|\frac{(x+2)(2-5x)}{x+2} - 12| = |(2-5x)-12| = |-5x-10| = 5|x+2| < 5\delta = \varepsilon$ whenever $0 < |x+2| < \delta$, as desired.

1g. Suppose $\varepsilon > 0$. Choose $\delta = \frac{\varepsilon}{10}$. Then $|\frac{10x^2+33x+9}{x+3} - (-27)| = |\frac{(10x+3)(x+3)}{x+3} + 27| = (10x+3)+27| = |10x+30| = 10|x+3| < 10\delta = \varepsilon$ whenever $0 < |x+3| < \delta$, as desired.

1h. Suppose $\varepsilon > 0$. Choose $\delta = \frac{\varepsilon}{6}$. Then $|\frac{x-6x^2}{x} - 1| = |\frac{x(1-6x)}{x} - 1| = |(1-6x)-1| = |-6x| = 6|x| < 6\delta = \varepsilon$ whenever $0 < |x-0| < \delta$, as desired.