No notes, books, electronic devices, or outside materials of any kind.

Read each problem carefully and simplify your answers. Unless otherwise indicated, supporting work will be required on every problem worth more than 2 points.

1a (12 pts). Compute the determinant of $F = \begin{bmatrix} 7 & -1 & 8 & 2 & 4 \\ 11 & 0 & 3 & -4 & -4 \\ 2 & 1 & -1 & 5 & 5 \\ 2 & 0 & 0 & 0 & 0 \\ -9 & 0 & 1 & 0 & 0 \end{bmatrix}$.

1b (3 pts). Is F invertible? Why?

2. Suppose $T : \mathbb{R}^k \to \mathbb{R}^\ell : \mathbf{x} \mapsto A\mathbf{x}$ for some matrix A.
(That is, the transformation T from $\mathbb{R}^k$ into $\mathbb{R}^\ell$ is given by the rule $T(\mathbf{x}) = A\mathbf{x}$.)

a (3 pts). What is the size of A?

b (5 pts). If T maps $\mathbb{R}^k$ *onto* $\mathbb{R}^\ell$, what, if anything, can you conclude about k and ℓ, and why?

3 (6 pts). If $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ are vectors in a vector space V, define their span. That is, explain what is meant by the symbol $\operatorname{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.

4 (10 pts). Answer either a. or b. Clearly indicate which part you're answering.

a. Let S be the transformation from $\mathbb{R}^2$ into $\mathbb{R}^2$ defined by $S\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - x_1x_2 \\ 2x_2 + x_1x_2 \end{bmatrix}$. Show that S is *not* linear.

b. Let H be the subset of all vectors $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ in $\mathbb{R}^2$ that satisfy $x_1x_2 - 2x_1 = 0$. Show that H is *not* a subspace of $\mathbb{R}^2$.

5a (17 pts). Apply the Gram-Schmidt process to the three vectors $\begin{bmatrix} 1 \\ 1 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -5 \\ 1 \\ 3 \end{bmatrix}$.

5b (5 pts). Find an orthonormal basis for $\operatorname{Span}\left\{\begin{bmatrix} 1 \\ 1 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -5 \\ 1 \\ 3 \end{bmatrix}\right\}$

6. Let $G = \begin{bmatrix} 1 & 2 \\ 1 & 2 \\ 2 & 1 \\ 0 & 1 \end{bmatrix}$ and $\mathbf{z} = \begin{bmatrix} 4 \\ 6 \\ -2 \\ 8 \end{bmatrix}$.

a (17 pts). Find the least-squares solution $\hat{\mathbf{x}}$ to $G\mathbf{x} = \mathbf{z}$.

b (5 pts). Find the projection of $\mathbf{z}$ onto $\operatorname{Col} G$.

c (5 pts). Find the distance from $\mathbf{z}$ to $\operatorname{Col} G$.

7. Let $E = \begin{bmatrix} 1 & 0 & 1 & -2 & -7 \\ 3 & 1 & 5 & -6 & -20 \\ 0 & -1 & -2 & 1 & 2 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} -3 \\ -6 \\ -1 \end{bmatrix}$.

a (18 pts). Find the general solution (if it exists) to $E\mathbf{x} = \mathbf{b}$. State your answer in parametric vector form.

b (4 pts). State the general solution to $E\mathbf{x} = \mathbf{0}$ in parametric vector form.

c (4 pts). Give a basis for the column space of E.

d (2 pts). What is the rank of E?

8 (21 pts). Find Z^{-1} if $Z = \begin{bmatrix} -2 & -5 & -4 \\ 3 & 7 & 8 \\ 1 & 2 & 3 \end{bmatrix}$.

9. Suppose $V^{-1} = \begin{bmatrix} 5 & -1 \\ 3 & 2 \end{bmatrix}$.

a (3 pts). Find $\mathbf{x}$ if $V\mathbf{x} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

b (5 pts). Solve for the matrix Y if $YV = \begin{bmatrix} 1 & 0 \\ 3 & -1 \end{bmatrix}$.

c (5 pts). Find $(VU)^{-1}$ if $U^{-1} = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$.

10 (5 pts). Suppose that $\mathbf{u}_1$, $\mathbf{u}_2$, $\mathbf{u}_3$, and $\mathbf{u}_4$ are four vectors in $\mathbb{R}^4$, and that the equation $x_1\mathbf{u}_1 + x_2\mathbf{u}_2 + x_3\mathbf{u}_3 + x_4\mathbf{u}_4 = \begin{bmatrix} 1 & -1 & 0 & 3 \end{bmatrix}^T$ has no solution.
How many solutions are there to $x_1\mathbf{u}_1 + x_2\mathbf{u}_2 + x_3\mathbf{u}_3 + x_4\mathbf{u}_4 = \mathbf{0}$, and why?

11 (17 pts). Use the given matrices to find the following, if they exist.

$$A = \begin{bmatrix} 1 & 4 & 0 \\ 0 & -1 & 2 \\ 0 & 0 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 3 & -1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 3 \end{bmatrix} \qquad \mathbf{x} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

Work in the space below and label your answers.

a. AB b. B^TA^T c. $\mathbf{x}^T\mathbf{x}$ d. $A\mathbf{x}$ e. $\mathbf{x}^TA^TA\mathbf{x}$

12 (8 pts). Let $T : \mathbb{R}^2 \to \mathbb{P}_2 : \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \mapsto x_1t(1-t) + x_2t(2-t)$.
Find the matrix for T relative to the standard bases for $\mathbb{R}^2$ and $\mathbb{P}_2$.

13 (5 pts). Suppose $\{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3, \mathbf{w}_4\}$ is a linearly independent set of vectors in the vector space W. What, if anything, can you conclude about the dimension of W, and why?

14 (6 pts). $\mathcal{B} = \left\{ \begin{bmatrix} -1 & 2 & -1 \end{bmatrix}^T, \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T, \begin{bmatrix} -1 & 0 & 1 \end{bmatrix}^T \right\}$ is an orthogonal basis for $\mathbb{R}^3$. Find the coordinates of $\mathbf{x} = \begin{bmatrix} 2 & -1 & 1 \end{bmatrix}^T$ relative to $\mathcal{B}$.

15 (9 pts). $\lambda = 3$ is an eigenvalues of the matrix $\begin{bmatrix} 9 & -4 & -4 \\ 6 & -1 & -4 \\ 6 & -4 & -1 \end{bmatrix}$.

Find a basis of the associated eigenspace.

1a. $\begin{vmatrix} 7 & -1 & 8 & 2 & 4 \\ 11 & 0 & 3 & -4 & -4 \\ 2 & 1 & -1 & 5 & 5 \\ 2 & 0 & 0 & 0 & 0 \\ 9 & 0 & 1 & 0 & 0 \end{vmatrix} = -2 \begin{vmatrix} -1 & 8 & 2 & 4 \\ 0 & 3 & -4 & -4 \\ 1 & -1 & 5 & 5 \\ 0 & 1 & 0 & 0 \end{vmatrix} = -2(1) \begin{vmatrix} -1 & 2 & 4 \\ 0 & -4 & -4 \\ 1 & 5 & 5 \end{vmatrix} = -2(-4) \begin{vmatrix} -1 & 2 & 4 \\ 0 & 1 & 1 \\ 1 & 5 & 5 \end{vmatrix}$.

now $r_3 \leftarrow r_3 - 5r_2$, then $r_1 \leftarrow r_1 + r_3$: $= +8 \begin{vmatrix} 0 & 2 & 4 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{vmatrix} = 8 \begin{vmatrix} 2 & 4 \\ 1 & 1 \end{vmatrix} = 8(2-4) = -16.$

1b. F <u>is</u> invertible, because $\det F \neq 0$.

2a. x is $k \times 1$, Ax is $\ell \times 1$; A must be $\ell \times k$.
(in alternate form of this question, $T : \mathbb{R}^\ell \to \mathbb{R}^k$. size $A = k \times \ell$)

2b. onto means columns of A span $\mathbb{R}^\ell$. # columns must $\geq \ell$, so $k \geq \ell$. (alt. form, $\ell \geq k$)

3. Span$\{v_1, v_2, v_3\}$ is the collection of all linear combinations of v_1, v_2, v_3. (In symbols, $\{x_1v_1 + x_2v_2 + x_3v_3 : x_1, x_2, x_3 \in \mathbb{R}\}$.

4a. $S\binom{1}{0} = \binom{1}{0}$, $S\binom{0}{1} = \binom{0}{2}$. But $S\binom{1}{1} = \binom{0}{3} \neq S\binom{1}{0} + S\binom{0}{1}$.
Because it isn't additive, S is <u>not</u> linear.

4b. $\binom{0}{1}$ is in H, because $0\cdot 1 - 2\cdot 0 = 0$.
$\binom{1}{2}$ is in H, because $1\cdot 2 - 2\cdot 1 = 0$. But their sum, $\binom{1}{3}$, is not in H, because $1\cdot 3 - 2\cdot 1 \neq 0$. Because it is not closed under addition, H is <u>not</u> a vector space.

5. $x_1 = \begin{pmatrix} 1 \\ 1 \\ -2 \\ 0 \end{pmatrix}$, $x_2 = \begin{pmatrix} 1 \\ 0 \\ -1 \\ 1 \end{pmatrix}$, $x_3 = \begin{pmatrix} 1 \\ -5 \\ 1 \\ 3 \end{pmatrix}$.

$u_1 = x_1 = \begin{pmatrix} 1 \\ 1 \\ -2 \\ 0 \end{pmatrix}$. $u_2 = x_2 - \operatorname{proj}_{\operatorname{span} u_1} x_2 = x_2 - \frac{x_2 \cdot u_1}{u_1 \cdot u_1} u_1$

$= \begin{pmatrix} 1 \\ 0 \\ -1 \\ 1 \end{pmatrix} - \frac{3}{6} \begin{pmatrix} 1 \\ 1 \\ -2 \\ 0 \end{pmatrix} = \begin{pmatrix} 1/2 \\ -1/2 \\ 0 \\ 1 \end{pmatrix}$. $\begin{pmatrix} 1 \\ -1 \\ 0 \\ 2 \end{pmatrix}$ will also work. (Check $u_1 \cdot u_2 = 0$ ✓)

$u_3 = x_3 - \operatorname{proj}_{\operatorname{span} u_1, u_2} x_3 = x_3 - \frac{x_3 \cdot u_1}{u_1 \cdot u_1} u_1 - \frac{x_3 \cdot u_2}{u_2 \cdot u_2}$

$= \begin{pmatrix} 1 \\ -5 \\ 1 \\ 3 \end{pmatrix} - \frac{-6}{6} \begin{pmatrix} 1 \\ 1 \\ -2 \\ 0 \end{pmatrix} - \frac{12}{6} \begin{pmatrix} 1 \\ -1 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ -2 \\ -1 \\ -1 \end{pmatrix}$. (Check $u_3 \cdot u_2 = u_3 \cdot u_1 = 0$ ✓)

5-alt. form. $x_1 = \begin{pmatrix}1\\0\\-1\\1\end{pmatrix}$, $x_2 = \begin{pmatrix}1\\1\\-2\\0\end{pmatrix}$, $x_3 = \begin{pmatrix}1\\-5\\1\\3\end{pmatrix}$.

$u_1 = x_1$; $\quad u_2 = x_2 - \frac{x_2 \cdot u_1}{u_1 \cdot u_1} = \begin{pmatrix}1\\1\\-2\\0\end{pmatrix} - \frac{3}{3}\begin{pmatrix}1\\0\\-1\\1\end{pmatrix} = \begin{pmatrix}0\\1\\-1\\-1\end{pmatrix}$.

$u_3 = x_3 - \frac{x_3 \cdot u_1}{u_1 \cdot u_1} u_1 - \frac{x_3 \cdot u_2}{u_2 \cdot u_2} u_2 = \begin{pmatrix}1\\-5\\1\\3\end{pmatrix} - \frac{3}{3}\begin{pmatrix}1\\0\\-1\\1\end{pmatrix} - \frac{-9}{3}\begin{pmatrix}0\\1\\-1\\-1\end{pmatrix} = \begin{pmatrix}0\\-2\\-1\\-1\end{pmatrix}$.

5b. Normalize the orthogonal basis found in 5a:

$\left\{ \frac{1}{\sqrt{6}}\begin{pmatrix}1\\1\\-2\\0\end{pmatrix}, \frac{1}{\sqrt{6}}\begin{pmatrix}1\\-1\\0\\2\end{pmatrix}, \frac{1}{\sqrt{6}}\begin{pmatrix}0\\-2\\-1\\-1\end{pmatrix} \right\}$ $\left(\text{alt: } \left\{ \frac{1}{\sqrt{3}}\begin{pmatrix}1\\0\\-1\\1\end{pmatrix}, \frac{1}{\sqrt{3}}\begin{pmatrix}0\\1\\-1\\-1\end{pmatrix}, \frac{1}{\sqrt{6}}\begin{pmatrix}0\\-2\\-1\\-1\end{pmatrix} \right\}.\right)$

6a. $\hat{x}$ is sol'n to $G^TGx = G^Tz$, $G^TG = \begin{pmatrix}6 & 6\\6 & 10\end{pmatrix}$. $G^Tz = \begin{pmatrix}6\\26\end{pmatrix}$. Augment.

$\begin{pmatrix}6 & 6 & 6\\6 & 10 & 26\end{pmatrix} \sim \begin{pmatrix}1 & 1 & 1\\0 & 4 & 20\end{pmatrix} \sim \begin{pmatrix}1 & 1 & 1\\0 & 1 & 5\end{pmatrix} \sim \begin{pmatrix}1 & 0 & -4\\0 & 1 & 5\end{pmatrix}$. $\hat{x} = \begin{pmatrix}-4\\5\end{pmatrix}$.

(alt form, $G^Tz = \begin{pmatrix}-6\\-26\end{pmatrix}$. $\hat{x} = \begin{pmatrix}4\\-5\end{pmatrix}$.)

6b. $\hat{z} = G\hat{x} = \begin{pmatrix}1 & 2\\1 & 2\\2 & 1\\0 & 1\end{pmatrix}\begin{pmatrix}-4\\5\end{pmatrix} = \begin{pmatrix}6\\6\\-3\\5\end{pmatrix}$. $\left(\text{alt. } \hat{z} = \begin{pmatrix}1 & 2\\1 & 2\\2 & 1\\0 & 1\end{pmatrix}\begin{pmatrix}4\\-5\end{pmatrix} = \begin{pmatrix}-6\\-6\\3\\-5\end{pmatrix}.\right)$

6c. distance $= \|z - \hat{z}\| = \left\| \begin{pmatrix}4\\6\\-2\\8\end{pmatrix} - \begin{pmatrix}6\\6\\-3\\5\end{pmatrix} \right\| = \left\| \begin{matrix}-2\\0\\1\\3\end{matrix} \right\| = \sqrt{14}$.

$\left(\text{alt. } \|z - \hat{z}\| = \left\| \begin{pmatrix}-8\\-6\\4\\-2\end{pmatrix} - \begin{pmatrix}-6\\-6\\3\\-5\end{pmatrix} \right\| = \left\| \begin{matrix}-2\\0\\1\\3\end{matrix} \right\| = \sqrt{14}.\right)$

7a $(E \,\vdots\, b) = \begin{array}{c} 1 \\ \begin{pmatrix}1 & 0 & 1 & -2 & -7 & -3\\3 & 1 & 5 & -6 & -20 & -6\\0 & -1 & -2 & 1 & 2 & -1\end{pmatrix}\end{array} \sim \begin{array}{c} 2 \\ \begin{pmatrix}1 & 0 & 1 & -2 & -7 & -3\\0 & 1 & 2 & 0 & 1 & 3\\0 & -1 & -2 & 1 & 2 & -1\end{pmatrix}\end{array}$

$\begin{array}{c} 3 \\ \begin{pmatrix}1 & 0 & 1 & -2 & -7 & -3\\0 & 1 & 2 & 0 & 1 & 3\\0 & 0 & 0 & 1 & 3 & 2\end{pmatrix}\end{array} \sim \begin{array}{c} 4 \\ \begin{pmatrix}1 & 0 & 1 & 0 & -1 & 1\\0 & 1 & 2 & 0 & 1 & 3\\0 & 0 & 0 & 1 & 3 & 2\end{pmatrix}\end{array} \Rightarrow$

$x_1 = 1 - x_3 + x_5$
$x_2 = 3 - 2x_3 - x_5$
$x_4 = 2 - 3x_5$
x_3, x_5 free.

$x = \begin{pmatrix}1\\3\\0\\2\\0\end{pmatrix} + x_3\begin{pmatrix}-1\\-2\\1\\0\\0\end{pmatrix} + x_5\begin{pmatrix}1\\-1\\0\\-3\\1\end{pmatrix}$

alt form: $b = \begin{pmatrix}-4\\-9\\-2\end{pmatrix}$. Last col of $(E \,\vdots\, b)$ becomes ...

① $\begin{pmatrix}-4\\-9\\-2\end{pmatrix}$ ② $\begin{pmatrix}-4\\3\\-2\end{pmatrix}$ ③ $\begin{pmatrix}-4\\3\\1\end{pmatrix}$ ④ $\begin{pmatrix}-2\\3\\1\end{pmatrix}$. $x = \begin{pmatrix}-2\\3\\0\\1\\0\end{pmatrix} + x_3\begin{pmatrix}-1\\-2\\1\\0\\0\end{pmatrix} + x_5\begin{pmatrix}1\\-1\\0\\-3\\1\end{pmatrix}$.

7b. $x_3\begin{pmatrix}-1\\-2\\1\\0\\0\end{pmatrix} + x_5\begin{pmatrix}1\\-1\\0\\-3\\1\end{pmatrix}$. 7c. pivot cols of $\underline{E}$ (1st, 2nd, 4th) $= \left\{\begin{pmatrix}1\\3\\0\end{pmatrix}, \begin{pmatrix}0\\1\\-1\end{pmatrix}, \begin{pmatrix}-2\\-6\\1\end{pmatrix}\right\}$

7d. rank = dim Col E = 3.

8. $\left(\begin{array}{ccc|ccc}-2&-5&-4&1&0&0\\3&7&8&0&1&0\\1&2&3&0&0&1\end{array}\right) \sim \left(\begin{array}{ccc|ccc}1&2&3&0&0&1\\-2&-5&-4&1&0&0\\3&7&8&0&1&0\end{array}\right) \sim \left(\begin{array}{ccc|ccc}1&2&3&0&0&1\\0&-1&2&1&0&2\\0&1&-1&0&1&-3\end{array}\right) \sim \left(\begin{array}{ccc|ccc}1&2&3&0&0&1\\0&+1&-2&-1&0&-2\\0&0&1&1&1&-1\end{array}\right)$

$\sim \left(\begin{array}{ccc|ccc}1&2&0&-3&-3&4\\0&1&0&1&2&-4\\0&0&1&1&1&-1\end{array}\right) \sim \left(\begin{array}{ccc|ccc}1&0&0&-5&-7&+12\\0&1&0&1&2&-4\\0&0&1&1&1&-1\end{array}\right)$. $Z^{-1} = \begin{pmatrix}-5&-7&12\\1&2&-4\\1&1&-1\end{pmatrix}$.

(alt form:

$\left(\begin{array}{ccc|ccc}3&7&8&1&0&0\\-2&-5&-4&0&1&0\\1&2&3&0&0&1\end{array}\right) \sim \left(\begin{array}{ccc|ccc}1&2&3&\boxed{0\;0}&1\\-2&-5&-4&\boxed{0\;1}&0\\3&7&8&\boxed{1\;0}&0\end{array}\right)$ Now, in solution ↑ interchange columns 4 and 5. Result: $Z^{-1} = \begin{pmatrix}-7&-5&12\\2&1&-4\\1&1&-1\end{pmatrix}$.

9a. $x = V^{-1}\begin{pmatrix}1\\-1\end{pmatrix} = \begin{pmatrix}5&-1\\3&2\end{pmatrix}\begin{pmatrix}1\\-1\end{pmatrix} = \begin{pmatrix}6\\1\end{pmatrix}$. (alt: $V^{-1}\begin{pmatrix}1\\-1\end{pmatrix} = \begin{pmatrix}5&1\\3&-2\end{pmatrix}\begin{pmatrix}1\\-1\end{pmatrix} = \begin{pmatrix}4\\5\end{pmatrix}$.)

9b. $y = \begin{pmatrix}1&0\\3&-1\end{pmatrix}V^{-1} = \begin{pmatrix}1&0\\3&-1\end{pmatrix}\begin{pmatrix}5&-1\\3&2\end{pmatrix} = \begin{pmatrix}5&-1\\12&-5\end{pmatrix}$. (alt $y = \begin{pmatrix}1&0\\3&-1\end{pmatrix}\begin{pmatrix}5&1\\3&-2\end{pmatrix} = \begin{pmatrix}5&1\\12&5\end{pmatrix}$).

9c. $(VU)^{-1} = U^{-1}V^{-1} = \begin{pmatrix}1&2\\-1&4\end{pmatrix}\begin{pmatrix}5&-1\\3&2\end{pmatrix} = \begin{pmatrix}11&3\\7&9\end{pmatrix}$. (alt $\begin{pmatrix}1&2\\-1&4\end{pmatrix}\begin{pmatrix}5&1\\3&-2\end{pmatrix} = \begin{pmatrix}11&-3\\7&-9\end{pmatrix}$.)

10. Let U be the matrix $[u_1\ u_2\ u_3\ u_4]$. Since $Ux = \begin{pmatrix}1\\-1\\9\\3\end{pmatrix}$ has no solution, E.F.(U) must contain a row of zeros. U cannot have a pivot in every column,
$Ux = 0$ has at least one solution, since $U0 = 0$. The presence of a free variable $\Rightarrow$ $Ux = 0$ has <u>infinitely many</u> solutions.

11.a. $AB = \begin{pmatrix}3&3&8\\0&-1&4\\0&0&9\end{pmatrix}$ b. $B^TA^T = (AB)^T = \begin{pmatrix}3&0&0\\3&-1&0\\8&4&9\end{pmatrix}$. c. $x^Tx = x\cdot x = 2$.

d. $Ax = \begin{pmatrix}1\\-2\\-3\end{pmatrix}$. e $x^TA^TAx = (Ax)^TAx = (Ax)\cdot(Ax) = 1+4+9 = 14$.

12. $T\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = x_1(0+t-t^2) + x_2(0+2t-t^2)$, so

$$M\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = x_1\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} + x_2\begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 1 & 2 \\ -1 & -1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}. \quad M = \begin{pmatrix} 0 & 0 \\ 1 & 2 \\ -1 & -1 \end{pmatrix}.$$

13. Every linearly independent set in W can be expanded to a basis for W, so $\dim W \geq 4$. (alt, $\dim W \geq 3$.)

14. To express x as a linear combination of elements of $\mathcal{B}$, you could augment and row reduce, but because $\mathcal{B}$ is orthogonal, $x = \frac{x\cdot b_1}{b_1\cdot b_1} b_1 + \frac{x\cdot b_2}{b_2\cdot b_2} b_2 + \frac{x\cdot b_3}{b_3\cdot b_3} b_3$. Coordinates are

$$\left(\frac{x\cdot b_1}{b_1\cdot b_1}, \frac{x\cdot b_2}{b_2\cdot b_2}, \frac{x\cdot b_3}{b_3\cdot b_3}\right) = \left(\frac{-5}{6}, \frac{2}{3}, \frac{-1}{2}\right). \text{ (alt } \left(\frac{1}{6}, \frac{2}{3}, -\frac{3}{2}\right).)$$

15. $$A - 3I = \begin{pmatrix} 6 & -4 & -4 \\ 6 & -4 & -4 \\ 6 & -4 & -4 \end{pmatrix} \sim \begin{pmatrix} 1 & -2/3 & -2/3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad x = \begin{pmatrix} 2/3\, x_2 + 2/3\, x_3 \\ x_2 \\ x_3 \end{pmatrix}$$

$$= x_2\begin{pmatrix} 2/3 \\ 1 \\ 0 \end{pmatrix} + x_3\begin{pmatrix} 2/3 \\ 0 \\ 1 \end{pmatrix}. \quad \text{Basis} = \left\{\begin{pmatrix} 2/3 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2/3 \\ 0 \\ 1 \end{pmatrix}\right\}.$$