MATH 203-01 QUIZ 11 NOV 21, 2014

Let $W = \text{span}\{u_1, u_2, u_3\}$, where

$$u_1 = \begin{pmatrix} 1 \\ 1 \\ -1 \\ 0 \end{pmatrix}, \; u_2 = \begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix}, \text{ and } u_3 = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix}.$$

a. Find the projection of $x = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$ onto W.

b. Find a vector u_4 so that $\{u_1, u_2, u_3, u_4\}$ is an orthogonal basis for $\mathbb{R}^4$.

Solution.

a. $$\hat{x} = \frac{x \cdot u_1}{u_1 \cdot u_1} u_1 + \frac{x \cdot u_2}{u_2 \cdot u_2} u_2 + \frac{x \cdot u_3}{u_3 \cdot u_3} u_3$$

$$= \frac{0}{3}\begin{pmatrix} 1 \\ 1 \\ -1 \\ 0 \end{pmatrix} + \frac{1}{3}\begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix} + \frac{1}{3}\begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1/3 \\ 0 \\ 1/3 \\ 2/3 \end{pmatrix}.$$

b. Since $x - \hat{x} = \begin{pmatrix} -1/3 \\ 0 \\ -1/3 \\ 1/3 \end{pmatrix}$ is in $W^\perp$, can use any nonzero multiple of this for u_4, say $\begin{pmatrix} -1 \\ 0 \\ -1 \\ 1 \end{pmatrix}$.