MATH 203-01 QUIZ 9 OCT 31, 2014

Diagonalize $A = \begin{pmatrix} -5 & 6 \\ -3 & 4 \end{pmatrix}$, if possible. That is, Find an invertible P and diagonal D for which

$$A = PDP^{-1}.$$

(You are not required to find P^{-1}.)

Solution: Find A's eigenvalues:

$$|A - \lambda I| = \begin{vmatrix} -5-\lambda & 6 \\ -3 & 4-\lambda \end{vmatrix} = (-5-\lambda)(4-\lambda) - (-18) = \lambda^2 + \lambda - 2$$
$$= (\lambda+2)(\lambda-1).$$

eigenvalues are $\lambda = -2$ and 1.

A is diagonalizable because it has 2 $(=n)$ distinct eigenvalues.

Next find a basis for each eigenspace $= \text{Nul}(A - \lambda I)$.

$\lambda = 1$. $(A - I) = \begin{pmatrix} -6 & 6 \\ -3 & 3 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix}$. $\begin{matrix} x_1 = x_2 \\ x_2 = x_2 \end{matrix}$.

$$x = x_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix}. \text{ Basis} = \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\}.$$

$\lambda = -2$.

$A - (-2)I = \begin{pmatrix} -3 & 6 \\ -3 & 6 \end{pmatrix} \sim \begin{pmatrix} 1 & -2 \\ 0 & 0 \end{pmatrix}$. $\begin{matrix} x_1 = 2x_2 \\ x_2 = x_2 \end{matrix}$.

$$x = x_2 \begin{pmatrix} 2 \\ 1 \end{pmatrix}. \text{ Basis} = \left\{ \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right\}.$$

Conclusion: $P = \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}$, $D = \begin{pmatrix} 1 & 0 \\ 0 & -2 \end{pmatrix}$. done.

Note: any nonzero multiples of $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ would serve as the columns of P. Order of eigenvalues in D and eigenvectors in P must be the same.