MATH 220–02 (Kunkle), Quiz 4 Name: ______________________

10 pts, 10 minutes Sept 26, 2023

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1 (10 pts). Integrate: $\displaystyle\int \frac{7x^2+3x-4}{(x+3)(x^2+1)}\,dx$

1.(Source: 7.4.23)

Find the partial fraction decomposition of the integrand.

The degree of the numerator is strictly less than that of the denominator, so long division is not necessary. Look for constants A, B, and C for which

$$\frac{7x^2+3x-4}{(x+3)(x^2+1)} = \frac{A}{x+3} + \frac{Bx+C}{x^2+1}.$$

Multiply both sides by $(x+3)(x^2+1)$:

$$7x^2+3x-4 = A(x^2+1) + (Bx+C)(x+3).$$

Now do the same things to both sides to obtain three equations in the three unknowns A, B, and C. The easiest thing to do first is to evaluate at $x=-3$. After that, I chose to evaluate at $x=0$, and then equate the x^2 coefficients:

$$\begin{aligned}
x=-3:&\qquad 50 = A\cdot 10 &&\implies A=5\\
x=0:&\qquad -4 = A + C\cdot 3 &&\implies C=-3\\
x^2\text{-coefficient:}&\qquad 7 = A+B &&\implies B=2.
\end{aligned}$$

So, the PFD is

$$\frac{7x^2+3x-4}{(x+3)(x^2+1)} = \frac{5}{x+3} + \frac{2x-3}{x^2+1}$$

and the integral is

$$\begin{aligned}
\int\left(\frac{5}{x+3}+\frac{2x-3}{x^2+1}\right)dx &= 5\int\frac{1}{x+3}\,dx + \int\frac{2x}{x^2+1}\,dx - 3\int\frac{1}{x^2+1}\,dx\\
&= 5\int\frac{d(x+3)}{x+3} + \int\frac{d(x^2+1)}{x^2+1} - 3\int\frac{1}{x^2+1}\,dx\\
&= 5\ln|x+3| + \ln(x^2+1) - 3\tan^{-1}x + C.
\end{aligned}$$

(done)

Comment: If you equated the coefficients of x^2, x, and 1, you'd end up with the more difficult system of equations

$$\begin{aligned}
x=0:&\qquad -4 = A+3C\\
x\text{-coefficient:}&\qquad 3 = 3B+C\\
x^2\text{-coefficient:}&\qquad 7 = A+B
\end{aligned}$$