16.1: Vector fields.

A **vector field** is a function from $\mathbb{R}^2$ into $\mathbb{R}^2$ or from $\mathbb{R}^3$ into $\mathbb{R}^3$. Think of the inputs as points (x, y) in the plane (or points (x, y, z) in space) and the outputs as vectors in $\langle u, v\rangle$ in the plane (or vectors $\langle u, v, w\rangle$ in space).

Examples of vector fields

1. If f is a differentiable function of two or three variables, then its gradient ∇f is a vector field.

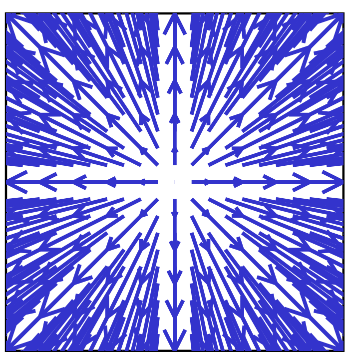
2. If a fluid is flowing across the plane or through space, then its velocity vector $\mathbf{v}$ at at each point (x, y) or (x, y, z) is a vector field.

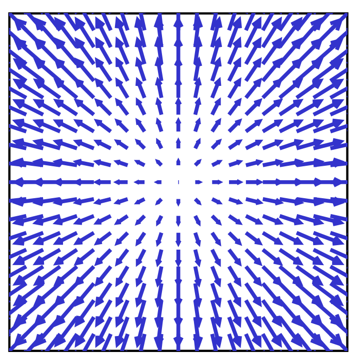
3. Anything that can be described as a force acting on an object depending on its position is a vector field. For instance, the gravitational force acting on an object of some fixed mass in space due to the presence of nearby masses is a vector field; its direction and magnitude depend on the object's position.

The graph of a vector field

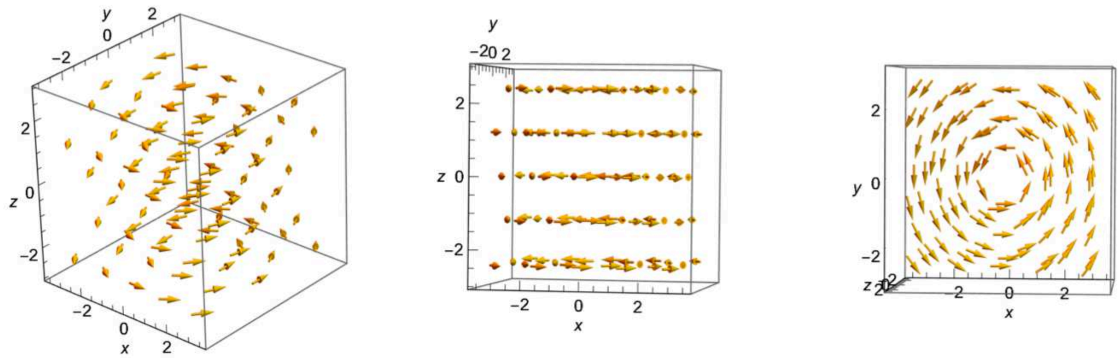
A **graph** of a vector field is a drawing of its values at selected points in the domain, drawn as vectors originating at those points. Graphing a vector field is best done by a computer, although drawing one by hand can be a worthwhile exercise.

16.1.re1. To make the graph of a vector field easier to read, it is common to draw the vectors at a reduced scale. Below are two graphs of the vector field $\mathbf{F}(x, y) = \langle x, y\rangle$. On the left, drawing $\langle x, y\rangle$ at its actual magnitude causes a lot of overlap. On the right, by graphing $c\langle x, y\rangle$ for some small scalar c, we can better see the behavior of $\langle x, y\rangle$.





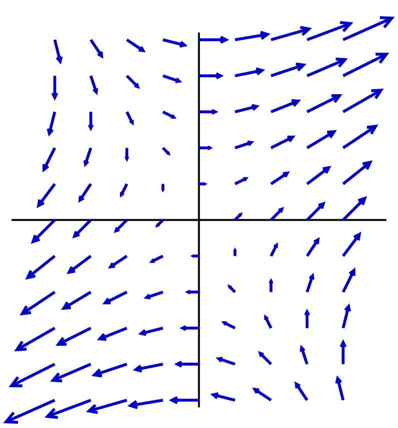
16.1.re2. Even when it is created by on a computer, the graph of a vector field in $\mathbb{R}^3$ can difficult to read without the ability to view it from different angles. Here's the same graph of $\mathbf{G}(x, y, z) = \langle -y, x, 0 \rangle$ seen from 3 different points of view.



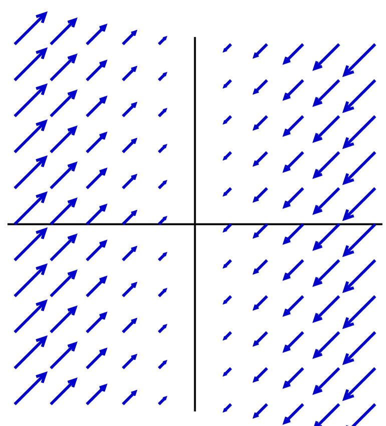
16.1.re3. Find the graph of the given vector field.

a. $\mathbf{A}(x, y) = \langle 1, 1 \rangle$ b. $\mathbf{B}(x, y) = \langle y, -x \rangle$ c. $\mathbf{C}(x, y) = \langle x + y, x \rangle$

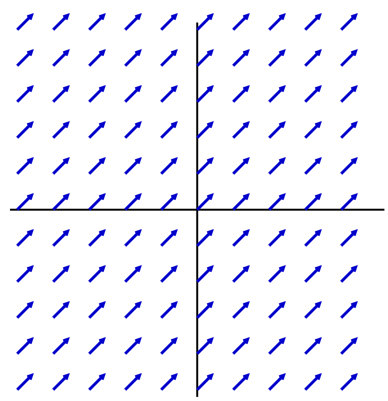
d. $\mathbf{D}(x, y) = \langle y, y \rangle$ e. $\mathbf{F}(x, y) = \langle -x, -x \rangle$ f. $\mathbf{M}(x, y) = \langle x + y, y \rangle$



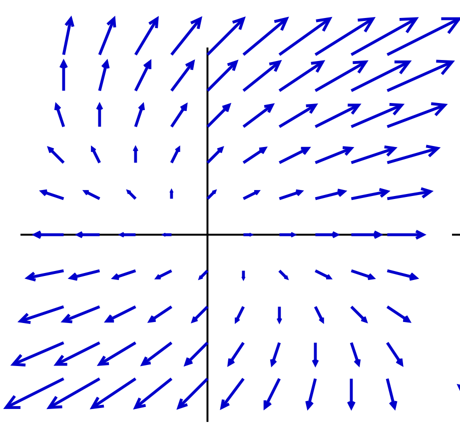
i



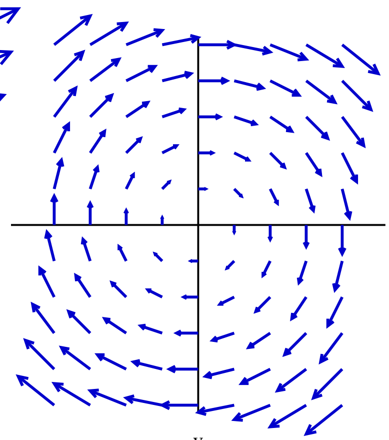
ii



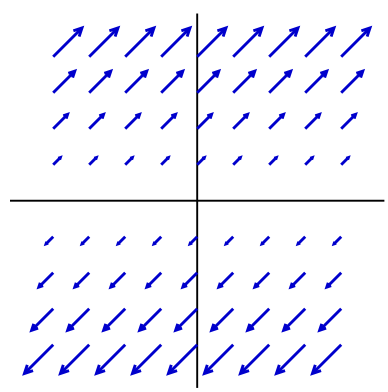
iii



iv

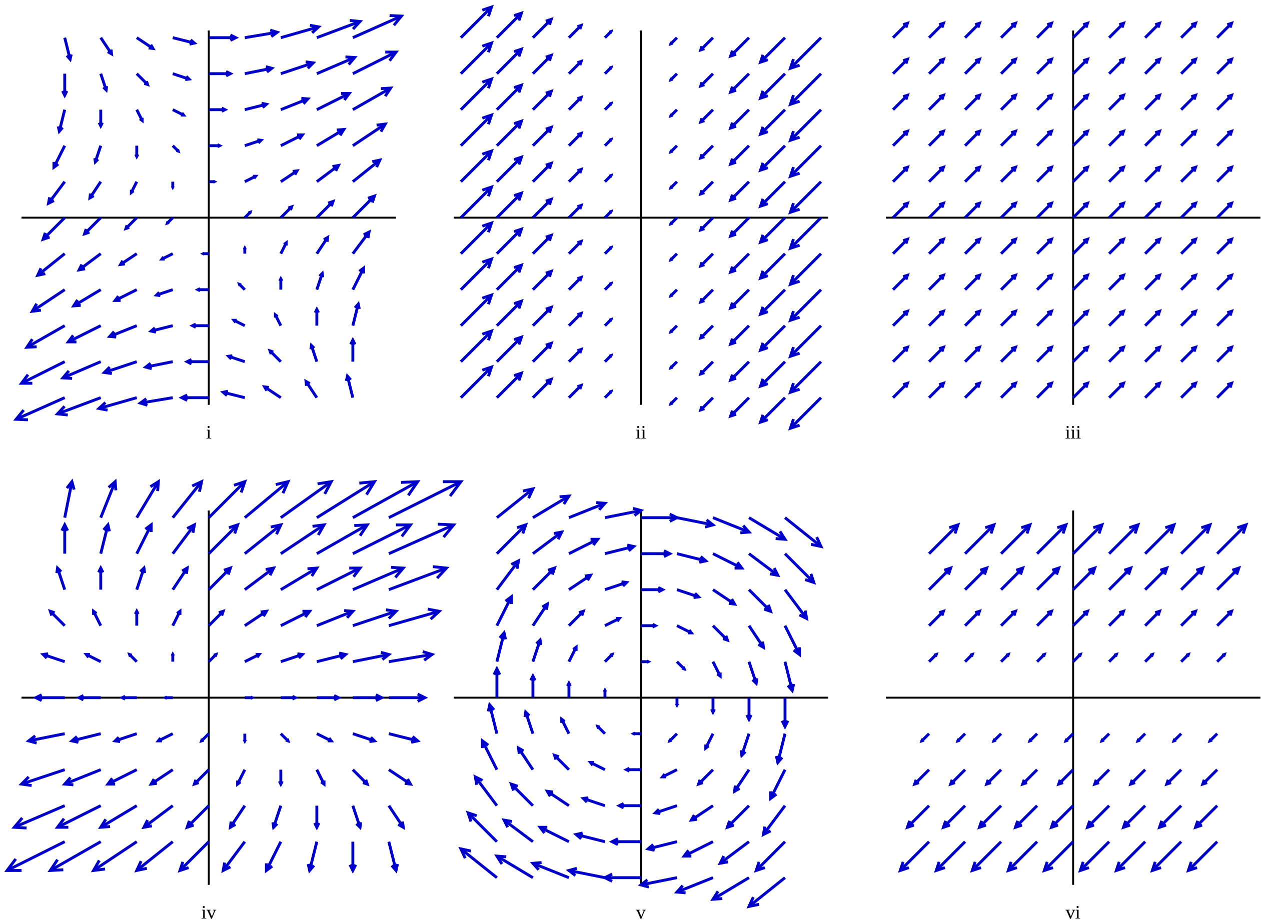


v



vi

16.1.re4. Find the vector fields graphed below.



a. $\langle y, 1\rangle$ b. $\langle -y, x\rangle$ c. $\langle x-y, x+y\rangle$

d. $\langle 1, -2\rangle$ e. $\langle -x, 2x\rangle$ f. $\langle y, -2y\rangle$

g. $\langle x+y, x-y\rangle$ h. $\langle x, -2x\rangle$ i. $\langle x, 1\rangle$

Answers

16.1.re3. ? 16.1.re4. ?