No notes, books, electronic devices, or outside materials of any kind.

Read each problem carefully and simplify your answers.

Unless otherwise indicated, supporting work will be required on every problem worth more than 2 points.

1(19 pts). Find the extreme values of ye^x subject to the constraint $x^2 + 2y^2 = 2$.

2(12 pts). Evaluate the double integral $\iint_R x\cos(xy)\,dA$ where $R = [0,1] \times [0,\pi]$

3(12 pts). Evaluate the triple integral $\int_0^3 \int_z^{2z} \int_0^{\ln y} ye^{-x}\,dx\,dy\,dz$.

4(9 pts). Find the volume of the region beneath the surface $z = 1 + \frac{2}{3}x^{3/2} - \frac{2}{3}y^{3/2}$ above the square $0 \le x \le 1$, $0 \le y \le 1$.

5(13 pts). Find the area of the surface $z = 1 + \frac{2}{3}x^{3/2} - \frac{2}{3}y^{3/2}$ above the square $0 \le x \le 1$, $0 \le y \le 1$.

6(9 pts). Evaluate the double integral of e^{x^2} over the triangle bounded by the lines $y = 0$, $x = 1$ and $y = 2x$.

7(13 pts). Evaluate the double integral of $\sqrt{2 + x^2 + y^2}$ over the region in the first quadrant bounded by the lines $y = 0$ and $y = x$ and the circle $x^2 + y^2 = 2$.

8(13 pts). Let R be the tetrahedron in the first octant bounded by the coordinate planes and the plane $x + 2y + 3z = 6$. Write the volume of R as an iterated triple integral in the order indicated, but **do not evaluate**:

a. $\iiint dx\,dy\,dz$

b. $\iiint dz\,dx\,dy$

1.(Source: 14.8.6) Set $f(x,y)=ye^x$ and $g(x,y)=x^2+2y^2$. The extrema of f along $g=2$ can only occur at those points at which

$$\mathbf{0}=\nabla f\times\nabla g=\langle ye^x,e^x,0\rangle\times\langle 2x,4y,0\rangle$$
$$=2e^x\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\ y&1&0\\ x&2y&0\end{vmatrix}=2e^x\langle 0,0,2y^2-x\rangle.$$

Since e^x is never zero, $2y^2=x$ at the critical points. Substituting this into $g=2$ gives $x^2+x=2$, whose solutions are $x=-2$ and $x=1$. Since $x=2y^2$ must be nonnegative, we ignore $x=-2$ and find that the only critical points are $x=1, y=\pm\sqrt{\frac{1}{2}}$. Evaluate $f(x,y)$ at these to find the max $=e\sqrt{\frac{1}{2}}$ and min $=-e\sqrt{\frac{1}{2}}$.

2.(Source: 15.1.33) The iterated integral is considerably easier if we integrate first with respect to y. (Integrating first with respect to x would require integration by parts.)

$$\int_0^1\int_0^\pi x\cos(xy)\,dy\,dx=\int_0^1 \sin(xy)\Big|_{y=0}^{y=\pi}dx=\int_0^1\big(\sin(\pi x)-\sin 0\big)\,dx$$
$$=\int_0^1\sin(\pi x)\,dx=-\frac{1}{\pi}\cos(\pi x)\Big|_0^1=-\frac{1}{\pi}(-1-1)=\frac{2}{\pi}$$

3.(Source: 15.6.5) Remembering that $e^{-\ln y}=e^{\ln y^{-1}}=\frac{1}{y}$,

$$\int_0^3\int_z^{2z}\int_0^{\ln y} ye^{-x}\,dx\,dy\,dz=\int_0^3\int_z^{2z} -ye^{-x}\Big|_{x=0}^{x=\ln y}dy\,dz=\int_0^3\int_z^{2z}(-ye^{-\ln y}+y)\,dy\,dz$$
$$=\int_0^3\int_z^{2z}\left(-y\frac{1}{y}+y\right)dy\,dz=\int_0^3\int_z^{2z}(-1+y)\,dy\,dz=\int_0^3(-y+\frac{1}{2}y^2)\Big|_{y=z}^{y=2z}dz$$
$$=\int_0^3\big(-2z+2z^2+z-\frac{1}{2}z^2\big)\,dz=\int_0^3\big(-z+\frac{3}{2}z^2\big)\,dz$$
$$=\big(-\frac{1}{2}z^2+\frac{1}{2}z^3\big)\Big|_0^3=-\frac{1}{2}3^2+\frac{1}{2}3^3=9.$$

4.(Source: 15.1.37-38) The volume is $\int_0^1\int_0^1(1+\frac{2}{3}x^{3/2}-\frac{2}{3}y^{3/2})\,dx\,dy$. The easiest way to calculate this integral is to first break it up:

$$\int_0^1\int_0^1 1\,dx\,dy+\int_0^1\int_0^1\frac{2}{3}x^{3/2}\,dx\,dy-\int_0^1\int_0^1\frac{2}{3}y^{3/2}\,dx\,dy,$$

then use $\int_0^1 C\,dx=C$ to perform the easiest integrations:

$$=1+\int_0^1\frac{2}{3}x^{3/2}\,dx-\int_0^1\frac{2}{3}y^{3/2}\,dy,$$

and then observe that $\int_0^1 \frac{2}{3}x^{3/2}\,dx = \int_0^1 \frac{2}{3}y^{3/2}\,dy$ (both equal $\frac{4}{15}$), so that the volume equals 1.

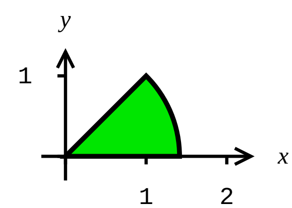
5.(Source: 15.5.8) The same surface as in Problem 4, but a different integral. $S = \iint dS = \int_0^1\int_0^1 \sqrt{1+z_x^2+z_y^2}\,dx\,dy = \int_0^1\int_0^1 \sqrt{1+(x^{1/2})^2+(-y^{1/2})^2}\,dx\,dy$

$$\begin{aligned}
&= \int_0^1\int_0^1 \sqrt{1+x+y}\,dx\,dy = \int_0^1 \frac{2}{3}(1+x+y)^{3/2}\Big|_0^1\,dy \\
&= \frac{2}{3}\int_0^1 \left((2+y)^{3/2}-(1+y)^{3/2}\right)dy = \frac{2}{3}\cdot\frac{2}{5}\left((2+y)^{5/2}-(1+y)^{5/2}\right)\Big|_0^1 \\
&= \frac{4}{15}(3^{5/2}-2^{5/2}-2^{5/2}+1) = \frac{4}{15}(3^{5/2}-2^{7/2}+1)
\end{aligned}$$

6.(Source: 15.2.16,51) Integrating e^{x^2} with respect to x will be impossible, so integrate first with respect to y:

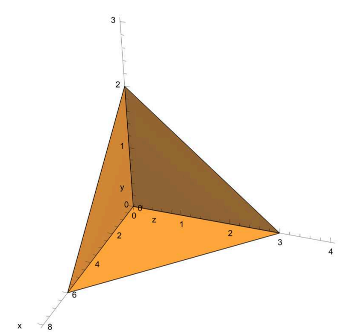
$$\int_0^1\int_0^{2x} e^{x^2}\,dy\,dx = \int_0^1 2xe^{x^2}\,dx = e^{x^2}\Big|_0^1 = e-1.$$

7.(Source: 15.3.8,9) See the figure for the region of integration. Both the integrand and the limits will be easier to write in polar coordinates. Remember that $dA = r\,dr\,d\theta$.



$$\int_0^{\pi/4}\int_0^{\sqrt{2}} r\sqrt{2+r^2}\,dr\,d\theta = \int_0^{\pi/4} \frac{1}{3}(2+r^2)^{3/2}\Big|_0^{\sqrt{2}}\,d\theta = \frac{\pi}{12}(4^{3/2}-2^{3/2})$$

8.(Source: 15.6.19,35) Drawing tip. To sketch the plane, plot its three intercepts: $(6,0,0)$, $(0,3,0)$, and $(0,0,2)$.



a. First find the yz-limits that describe the footprint of this solid in the yz plane. Then observe that, for every (y,z) in this footprint, x goes from 0 to the corresponding x-coordinate on the plane. The integral is

$$\int_0^2\int_0^{\frac{1}{2}(6-3z)}\int_0^{6-2z-3y} dx\,dy\,dz$$

b. For this part, find the xy-limits that describe the footprint of this solid in the xy plane. For every (x,y) in this footprint, z goes from 0 to the corresponding z-coordinate on the plane, and the integral is

$$\int_0^3\int_0^{6-2y}\int_0^{\frac{1}{3}(6-x-2y)} dz\,dx\,dy$$