No notes, books, electronic devices, or outside materials of any kind.

Read each problem carefully and simplify your answers.

Unless otherwise indicated, supporting work will be required on every problem worth more than 2 points.

You are expected to know the values of all trig functions at multiples of $\pi/4$ and of $\pi/6$.

1a(4 pts). Rewrite the equation $xy - z = 2$ in cylindrical coordinates.

1b(4 pts). Rewrite the equation $xy - z = 2$ in spherical coordinates.

1c(3 pts). Find cylindrical coordinates of the point $(x, y, z) = (\sqrt{2}, \sqrt{2}, -2\sqrt{3})$.

$r =$ ________ $\theta =$ ________ $z =$ ________

1d(3 pts). Find spherical coordinates of the point $(x, y, z) = (\sqrt{2}, \sqrt{2}, -2\sqrt{3})$.

$\rho =$ ________ $\phi =$ ________ $\theta =$ ________

2(8 pts). Compute a Riemann sum for the function $x + y$ on the rectangle $[-1, 1] \times [0, 1]$ using $m = n = 2$ sub-intervals in both the x- and y-direction and taking the sample points to be the midpoints of the sub-rectangles.

3(7 pts). Evaluate the double integral $\iint_{[-1,1]\times[0,1]} (x + y)\, dA$.

4(10 pts). Evaluate the double integral $\iint_G x^2\, dA$ where G is the triangle with vertices $(0, 0)$, $(1, 1)$, $(0, 3)$.

5(15 pts). Find the volume of the region inside both the cylinder $x^2 + y^2 = 9$ and the ellipsoid $4x^2 + 4y^2 + z^2 = 64$.

6(19 pts). Evaluate the triple integral $\iiint_D \sqrt{x^2 + y^2 + z^2}\, dV$ where D is the region above the cone $z = \sqrt{x^2 + y^2}$ and inside the sphere $x^2 + y^2 + z^2 = 4$.

7(11 pts). Find the surface area of the graph of $z = 1 + 2x^2 + 3y$ above the triangle in the xy-plane bounded by the lines $x = 0$, $y = 0$, and $x + 3y = 3$. Express your answer as an iterated integral, but **do not evaluate**.

8(16 pts). Let P be the region in the first octant bounded by the plane $x + 2y + 3z = 6$. Express $\iiint_P f(x, y, z)\, dV$ (for an unspecified function f) in the given order.

a. $\iiint f\, dx\, dy\, dz$

b. $\iiint f\, dy\, dz\, dx$

1a(4 pts).(Source: 15.7.9,10) $x = r\cos\theta$ and $y = r\sin\theta$, so the equation becomes

$$r^2\cos\theta\sin\theta - z = 2.$$

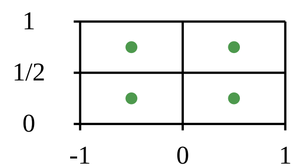
1b(4 pts).(Source: 15.8.9,10) $z = \rho\cos\phi$ and $r = \rho\sin\phi$, so the equation becomes

$$\rho^2\sin^2\phi\cos\theta\sin\theta - \rho\cos\phi = 2.$$

1c(3 pts).(Source: 15.7.3,4) $r = \sqrt{x^2+y^2} = 2$. The ray from the origin to the point $(\sqrt{2}, \sqrt{2})$ in the xy-plane make an angle $\frac{\pi}{4}$ with the positive x-axis, so $\theta = \frac{\pi}{4}$. z in cylindrical coordinates is the same as z in rectangular coordinates, so $z = -2\sqrt{3}$.

1d(3 pts).(Source: 15.7.3,4) $\rho = \sqrt{x^2+y^2+z^2} = 4$. The right triangle with sides $\rho = 4$, $r = 2$, and $z = -2\sqrt{3}$ is a $\frac{\pi}{6}$-$\frac{\pi}{3}$-$\frac{\pi}{2}$ with the $\frac{\pi}{3}$ angle between sides 2 and 4. $\phi = \frac{\pi}{2} + \frac{\pi}{3} = \frac{5\pi}{6}$.

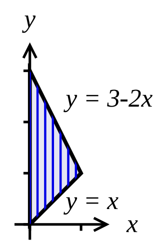
2(8 pts).(Source: 15.1.1) Solution: It helps to picture the subdivision of the rectangle:



The sample points (●) are at $x = \pm\frac{1}{2}$ and $y = \frac{1}{4}, \frac{3}{4}$. Dimensions of the sub-rectangles are $\Delta x = 1$ by $\Delta y = \frac{1}{2}$, and the Riemann sum is $1 \cdot \frac{1}{2} \cdot \left((-\frac{1}{2}+\frac{1}{4}) + (-\frac{1}{2}+\frac{3}{4}) + (\frac{1}{2}+\frac{1}{4}) + (\frac{1}{2}+\frac{3}{4})\right) = 1$.

3(7 pts).(Source: 15.1.16) Evaluate the double integral by iterated integration:
$\int_{-1}^{1}\int_0^1 (x+y)\,dy\,dx = \int_{-1}^{1}(xy+\frac{1}{2}y^2)\Big|_0^1\,dx = \int_{-1}^{1}(x+\frac{1}{2})\,dx = (\frac{1}{2}x^2+\frac{1}{2}x)\Big|_{-1}^{1} = 1.$

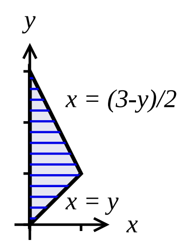
4(10 pts).(Source: 15.2.19) The line from $(0,0)$ to $(1,1)$ is $y = x$, and the line from $(1,1)$ to $(0,3)$ is $y = 3-2x$. The iterated integral is



$$\int_0^1\int_x^{3-2x} x^2\,dy\,dx = \int_0^1 (3-3x)x^2\,dx$$

$$= 3\int_0^1 (x^2-x^3)\,dx = 3(\frac{1}{3}x^3 - \frac{1}{4}x^4)\Big|_0^1 = \frac{1}{4}$$

Integrating in the other order will produce the same result but will require breaking the integral into two:



$$\int_0^1\int_0^y x^2\,dx\,dy + \int_1^3\int_0^{(3-y)/2} x^2\,dx\,dy$$

5(15 pts).(Source: 15.3.27,15.7.24) The ellipsoid is symmetric across the xy-plane, so the volume is double the volume above that plane. You can write the volume as either a triple integral in spherical coordinates ($dV = r\,dz\,dr\,d\theta$):

$$2\int_0^{2\pi}\int_0^3\int_0^{\sqrt{64-4r^2}} dz\,r\,dr\,d\theta$$

or a double integral in polar coordinates ($dA = r\,dr\,d\theta$):

$$2\int_0^{2\pi}\int_0^3 r(\sqrt{64-4r^2})\,dr\,d\theta.$$

After one integration, the triple integral becomes exactly this double integral. Proceeding,

$$2\int_0^{2\pi}\left(-\frac{1}{8}\cdot\frac{2}{3}(64-4r^2)^{3/2}\Big|_0^3\right)d\theta = \frac{\pi}{3}(64^{3/2}-28^{3/2}),$$

or, $\frac{\pi}{3}(512-28^{3/2}) = \frac{8\pi}{3}(64-t^{3/2})$.

6(19 pts).(Source: 15.8.25,26) In spherical coordinates, the cone is $\phi = \frac{\pi}{4}$, the sphere is $\rho = 2$, the integrand is ρ, and $dV = \rho^2 \sin\phi\,d\rho\,d\phi\,d\theta$, and so the triple integral is

$$\int_0^{2\pi}\int_0^{\pi/4}\int_0^2 \rho^3\sin\phi\,d\rho\,d\phi\,d\theta = \int_0^{2\pi}d\theta\int_0^{\pi/4}\sin\phi\,d\phi\int_0^2\rho^3\,d\rho$$
$$= 2\pi\cdot\left(-\cos\phi\Big|_0^{\pi/4}\right)\cdot\left(\frac{1}{4}\rho^4\Big|_0^2\right) = 2\pi\left(1-\frac{1}{\sqrt{2}}\right)\frac{1}{4}2^4,$$

or $4\pi(2-\sqrt{2})$.

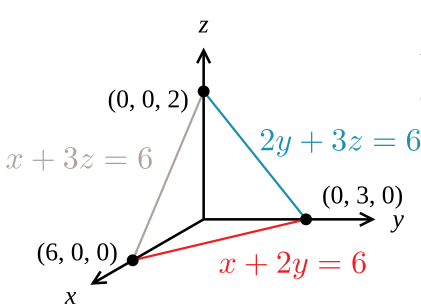
The integral is prohibitively difficult using cylindrical coordinates. Since the cone and sphere intersect at $x^2+y^2=2$, the integral starts as

$$\int_0^{2\pi}\int_0^{\sqrt{2}}\int_r^{\sqrt{4-r^2}} r\sqrt{r^2+z^2}\,dz\,dr\,d\theta$$

7(11 pts).(Source: 15.5.3,4) The surface area is the integral of $dS = \sqrt{1+z_x^2+z_y^2}\,dx\,dy = \sqrt{1+(4x)^2+3^2}\,dx\,dy$ over the triangle. This can be written either of two ways:

$$\int_0^1\int_0^{3-3y}\sqrt{16x^2+10}\,dx\,dy = \int_0^3\int_0^{1-\frac{1}{3}x}\sqrt{16x^2+10}\,dy\,dx.$$

8(16 pts).(Source: 15.6.19,27,32) Intercepts of $x+2y+3z=6$ are $(6,0,0)$, $(0,3,0)$, and $(0,0,2)$. The equations of the three lines joining these intercepts in the coordinate planes are obtained by setting one variable equal to 0 in the equation of the plane.



a. $$\int_0^2\int_0^{3-\frac{3}{2}z}\int_0^{6-2y-3z} f(x,y,z)\,dx\,dy\,dz$$

b. $$\int_0^6\int_0^{2-\frac{1}{3}x}\int_0^{3-\frac{1}{2}x-\frac{3}{2}z} f(x,y,z)\,dy\,dz\,dx$$