MATH 221–01 (Kunkle), Quiz 1 Name: ____________________

10 pts, 10 minutes Jan 20, 2022

---

1 (10 pts). Let

$$\mathbf{u} = \langle 3, -2, 1\rangle \qquad \mathbf{v} = \langle 0, 1, -2\rangle \qquad \mathbf{w} = \langle 2, 6, -3\rangle$$

Find the following or state that they do not exist. If any of your answers is zero, carefully distinguish between the scalar 0 and the vector $\mathbf{0}$.

a. $\mathbf{u} + 2\mathbf{v}$ b. $\mathbf{u} \cdot \mathbf{w}$ c. $\mathbf{w} \cdot \mathbf{w}$ d. the cosine of the angle between $\mathbf{u}$ and $\mathbf{w}$.
e. $(\mathbf{u} \cdot \mathbf{v}) \cdot \mathbf{w}$ f. $\mathbf{u} \times \mathbf{v}$ g. $\mathbf{v} \times \mathbf{u}$ h. $\mathbf{v} \cdot (\mathbf{v} \times \mathbf{w})$

1.(Source: 12.2.22, 12.3.1,12.3.5, 12.4.1, 12.4.13, 12.4.17)

a. $\mathbf{u} + 2\mathbf{v} = \langle 3, -2, 1\rangle + 2\langle 0, 1, -2\rangle = \langle 3, -2, 1\rangle + \langle 0, 2, -4\rangle = \langle 3, 0, -3\rangle$.

b. $\mathbf{u} \cdot \mathbf{w} = \langle 3, -2, 1\rangle \cdot \langle 2, 6, -3\rangle = (3)(2) + (-2)(6) + (1)(-3) = -9$.

c. $\mathbf{w} \cdot \mathbf{w} = \langle 2, 6, -3\rangle \cdot \langle 2, 6, -3\rangle = 2^2 + 6^2 + (-3)^2 = 49$.

d. $\cos\theta = \dfrac{\mathbf{u} \cdot \mathbf{w}}{|\mathbf{u}||\mathbf{w}|} = \dfrac{-9}{\sqrt{\mathbf{u} \cdot \mathbf{u}}\sqrt{\mathbf{w} \cdot \mathbf{w}}} = \dfrac{-9}{7\sqrt{14}}$.

e. Does not exist. $\mathbf{u} \cdot \mathbf{v}$ is a scalar, which cannot be dotted with a vector.

f. $$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -2 & 1 \\ 0 & 1 & -2 \end{vmatrix} = \mathbf{i}\begin{vmatrix} -2 & 1 \\ 1 & -2 \end{vmatrix} - \mathbf{j}\begin{vmatrix} 3 & 1 \\ 0 & -2 \end{vmatrix} + \mathbf{k}\begin{vmatrix} 3 & -2 \\ 0 & 1 \end{vmatrix}$$
$$= ((-2)^2 - 1^2)\mathbf{i} - (3)(-2)\mathbf{j} + (3)(1)\mathbf{k} = \langle 3, 6, 3\rangle$$

g. $\mathbf{v} \times \mathbf{u} = -(\mathbf{u} \times \mathbf{v}) = -\langle 3, 6, 3\rangle = \langle -3, -6, -3\rangle$

h. $\mathbf{v} \cdot (\mathbf{v} \times \mathbf{w}) =$ (the scalar) 0, since the vector $\mathbf{v} \times \mathbf{w}$ is orthogonal to $\mathbf{v}$ (and to $\mathbf{w}$.