MATH 221–01 (Kunkle), Quiz 2 | Name: ______________________
10 pts, 10 minutes | Jan 27, 2022

1a (8 pts). Find a parametric equation of the line tangent to the graph of

$$\mathbf{r}(t) = \sin t\,\mathbf{i} - 3\cos t\,\mathbf{j} + t^2\,\mathbf{k}$$

at the point corresponding to $t = \pi$.
1b (2 pts). Find the unit tangent $\mathbf{T}$ at the point corresponding to $t = \pi$ along the same curve.

Solution:
1a.(Source: 13.2.25) Differentiate:

$$\begin{aligned} \mathbf{r}(t) &= \sin t\,\mathbf{i} - 3\cos t\,\mathbf{j} + t^2\,\mathbf{k} \\ \frac{d\mathbf{r}}{dt} &= \cos t\,\mathbf{i} + 3\sin t\,\mathbf{j} + 2t\,\mathbf{k} \end{aligned}$$

At $t = \pi$,

$$\begin{aligned} \mathbf{r} &= 3\,\mathbf{j} + \pi^2\,\mathbf{k} = \langle 0, 3, \pi^2 \rangle \\ \frac{d\mathbf{r}}{dt} &= -\,\mathbf{i} + 2\pi\,\mathbf{k} = \langle -1, 0, 2\pi \rangle \end{aligned}$$

Now, as in section 12.5, the required tangent line is

$$x = -t \quad y = 3 \quad z = \pi^2 + 2\pi t.$$

(Compare with problem 12.5.2, for example.)
1b.(Source: 13.2.19) Normalize $\frac{d\mathbf{r}}{dt}$ to find the the unit tangent required in part c:

$$\mathbf{T} = \frac{\langle -1, 0, 2\pi \rangle}{|\langle -1, 0, 2\pi \rangle|} = \frac{1}{\sqrt{1 + 4\pi^2}} \langle -1, 0, 2\pi \rangle.$$

(done)