MATH 221–01 (Kunkle), Quiz 4 Name: ______________________________
10 pts, 10 minutes Feb 17, 2022

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1 (10 pts).
a. Find the maximum rate of change of $f(x,y,z) = \dfrac{x}{y-z}$ at the point $(1, \frac{1}{2}, 0)$, and the direction in which it occurs. Give the direction as a unit vector.
b. Find an equation of the plane tangent to the surface $\dfrac{x}{y-z} = 2$ at $(1, \frac{1}{2}, 0)$.

Solution: 1a. First calculate the gradient:

$$\nabla f = \langle f_x, f_y, f_z \rangle = \left\langle \frac{1}{y-z}, \frac{-x}{(y-z)^2}, \frac{x}{(y-z)^2} \right\rangle$$

At the point in question,

$$\begin{aligned} \nabla f(1, \tfrac{1}{2}, 0) &= \langle 2, -4, 4 \rangle = 2\langle 1, -2, 2 \rangle, \text{ and} \\ |\nabla f(1, \tfrac{1}{2}, 0)| &= 2\sqrt{9} = 6, \end{aligned}$$

so the greatest derivative of f at $(1, \frac{1}{2}, 0)$ is 6, and this occurs in the direction of the gradient. The direction of $\langle 2, -4, 4 \rangle$ is the same as that of $\langle 1, -2, 2 \rangle$, or $\frac{1}{3}\langle 1, -2, 2 \rangle$.

1b. The tangent plane passes through $(1, \frac{1}{2}, 0)$ and is normal to $\nabla f(1, \frac{1}{2}, 0)$, so the plane is given by the equation $\langle 1, -2, 2 \rangle \cdot \langle x - 1, y - \frac{1}{2}, z \rangle = 0$; that is,

$$\begin{aligned} x - 1 - 2(y - \frac{1}{2}) + 2z &= 0, \text{ or} \\ x - 2y + 2z &= 0. \end{aligned}$$

(done)