MATH 221–01 (Kunkle), Quiz 6 Name: ________________________
10 pts, 10 minutes March 17, 2022

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1 (10 pts). Find the surface area of the hyperbolic paraboloid $z = 2 + x^2 - y^2$ that lies inside the cylinder $x^2 + y^2 = 4$

1.(Source: 15.5.5, 15.3.34) $dA = \sqrt{1 + z_x^2 + z_y^2}\,dx\,dy = \sqrt{1 + 4x^2 + 4y^2}\,dx\,dy$. Switch to polar coordinates, in which both the integrand and the domain of integration—the circle centered at the origin of radius 2—are more easily written. Remember that

$$x^2 + y^2 = r^2 \qquad x = r\cos\theta \qquad y = r\sin\theta \qquad \frac{y}{x} = \tan\theta$$

and

$$dx\,dy = r\,dr\,d\theta.$$

The area is

$$\begin{aligned} A = \int dA &= \int_0^{2\pi}\int_0^2 \sqrt{1+4r^2}\,r\,dr\,d\theta \\ &= \int_0^{2\pi} \tfrac{1}{12}(1+4r^2)^{3/2}\Big|_0^2\,d\theta \\ &= \int_0^{2\pi} \tfrac{1}{12}(17^{3/2}-1)\,d\theta \\ &= \tfrac{1}{12}(17^{3/2}-1)(2\pi) \\ &= \tfrac{\pi}{6}(17^{3/2}-1) \end{aligned}$$

(done)