MATH 221–01 (Kunkle), Quiz 8 Name: _____
10 pts, 10 minutes April 7, 2022

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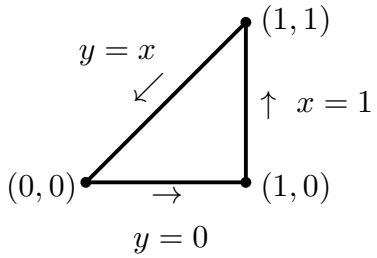
1 (10 pts). Find the work done by the force $\mathbf{F} = (e^{(x^2)} + y)\mathbf{i} + (\sqrt{\sin y} + xy)\mathbf{j}$ in moving a particle along straight lines from $(0,0)$ to $(1,0)$ to $(1,1)$ and then back to $(0,0)$ by first writing the work as a double integral over the interior of this triangular path.

Solution: 1(10 pts).(Source: 16.4.7,17) The work done by $\mathbf{F}$ is the line integral $\oint \mathbf{F} \cdot d\mathbf{r} = \oint P\,dx + Q\,dy$ around the triangle,



which Green's Theorem tells us is the same as the double integral of $Q_x - P_y$ over the interior of the triangle.

$$\begin{aligned}
\int_0^1 \int_0^x (Q_x - P_y)\,dy\,dx &= \int_0^1 \int_0^x (y-1)\,dy\,dx \\
&= \int_0^1 \frac{1}{2}(y-1)^2\Big|_0^x\,dx \\
&= \frac{1}{2}\int_0^1 \left((x-1)^2 - 1\right)dx \\
&= \frac{1}{2}\left(\frac{1}{3}(x-1)^3 - x\Big|_0^1\right) \\
&= \frac{1}{2}\left(-1 - \frac{1}{3}(-1)^3\right) \\
&= -\frac{1}{3}
\end{aligned}$$

(done)