

1 (10 pts). Let $\mathbf{u} = \langle 2, -3, 1 \rangle$ and $\mathbf{v} = \langle -1, -2, 1 \rangle$.

- Find the vector projection of \mathbf{v} onto \mathbf{u} .
 - Find the scalar projection of \mathbf{v} onto \mathbf{u} .
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Solution:

1a.(Source: 12.3.41-42)

$$\begin{aligned}\text{proj}_{\mathbf{u}} \mathbf{v} &= \frac{\mathbf{v} \cdot \mathbf{u}}{\mathbf{u} \cdot \mathbf{u}} \mathbf{u} = \frac{2 \cdot (-1) + (-3) \cdot (-2) + 1 \cdot 1}{2^2 + (-3)^2 + 1^2} \langle 2, -3, 1 \rangle \\ &= \frac{5}{14} \langle 2, -3, 1 \rangle, \text{ or } \left\langle \frac{5}{7}, \frac{-15}{14}, \frac{5}{14} \right\rangle.\end{aligned}$$

1b.(Source: 12.3.42)

$$\text{comp}_{\mathbf{u}} \mathbf{v} = \frac{\mathbf{v} \cdot \mathbf{u}}{|\mathbf{u}|} = \frac{5}{\sqrt{14}}.$$

The scalar projection is the same as $|\mathbf{v}| \cos \theta$, the signed length of the vector projection. In this case, the sign is positive, because the projection of \mathbf{v} onto \mathbf{u} points in the same direction as \mathbf{u} .