MATH 221–02 (Kunkle), Quiz 5 | Name: ____________________
10 pts, 10 minutes | Feb 29, 2024

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1 (10 pts). Calculate the double integral $\iint_R e^{x-y}\,dA$, where $R = [0,1] \times [-2,0]$.

1.(Source: 15.1.31) You can write the double integral as an iterated integral either

$$\int_{-2}^{0}\int_{0}^{1} e^{x-y}\,dx\,dy \quad \text{or} \quad \int_{0}^{1}\int_{-2}^{0} e^{x-y}\,dy\,dx.$$

The work is the about same either way. Here's the solution using the first.

$$\begin{aligned}\int_{-2}^{0}\left(\int_{0}^{1} e^{x-y}\,dx\right)dy &= \int_{-2}^{0}\left(e^{x-y}\Big|_{0}^{1}\right)dy = \int_{-2}^{0}\left(e^{1-y} - e^{-y}\right)dy \\ &= \left(-e^{1-y} + e^{-y}\right)\Big|_{-2}^{0} \\ &= (-e^{1} + e^{0}) - (-e^{3} + e^{2}) = e^{3} - e^{2} - e + 1.\end{aligned}$$

(done)

Here's something you can do whenever you're integrating an integrand of the form $f(x)g(y)$ over a rectangle.

Rewrite the integrand as a product.

$$\int_{-2}^{0}\int_{0}^{1} e^{x-y}\,dx\,dy = \int_{-2}^{0}\int_{0}^{1} e^{x}e^{-y}\,dx\,dy$$

Since e^{-y} is independent of x, we can factor it out of the dx-integral.

$$= \int_{-2}^{0} e^{-y}\left(\int_{0}^{1} e^{x}\,dx\right)dy$$

Now $\int_0^1 e^x\,dx$ is a constant, so we can factor it out of the dy-integral, and then calculate the two single integrals.

$$\begin{aligned}&= \left(\int_{0}^{1} e^{x}\,dx\right)\left(\int_{-2}^{0} e^{-y}\,dy\right) \\ &= \left(e^{x}\Big|_{0}^{1}\right)\left(-e^{-y}\Big|_{-2}^{0}\right) \\ &= (e-1)\left(-1+e^{2}\right) = e^{3} - e^{2} - e + 1.\end{aligned}$$

(done)