MATH 221–02 (Kunkle), Quiz 7 | Name: ______________________

10 pts, 10 minutes | Mar 28, 2024

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1 (10 pts). Evaluate the double integral $\iint_P (x-y)e^{x+y}\,dA$, where P is the rectangle bounded by the lines $x = y - 1$, $x = y + 2$, $y = -x$, and $y = -x + 3$.

1.(Source: 15.9.24) Let $u = x - y$ and $v = x + y$. Solve for x and y to obtain $x = \frac{1}{2}u + \frac{1}{2}v$ and $y = \frac{1}{2}v - \frac{1}{2}u$. Calculate the Jacobian:

$$\frac{\partial(x,y)}{\partial(u,v)} = \left\| \begin{matrix} x_u & x_v \\ y_u & y_v \end{matrix} \right\| = \left\| \begin{matrix} 1/2 & 1/2 \\ -1/2 & 1/2 \end{matrix} \right\| = |1/4 - (-1/4)| = 1/2$$

The integral is

$$\begin{aligned} \int_{-1}^{2} \int_{0}^{3} ue^v \frac{1}{2}\,dv\,du &= \frac{1}{2} \int_{-1}^{2} u\,du \int_{0}^{3} e^v\,dv \\ &= \frac{1}{2} \left(\frac{1}{2}u^2 \Big|_{-1}^{2} \right) \left(e^v \Big|_{0}^{3} \right) \\ &= \frac{3}{4}(e^3 - 1) \end{aligned}$$

(Since $\frac{\partial(x,y)}{\partial(u,v)}$ is a constant, it could also be calculated as the reciprocal of $\frac{\partial(u,v)}{\partial(x,y)}$, which avoids the work of solving for x and y in terms of u and v.)